Proceedings of the International Conference on Manufacturing Systems – ICMaS
Vol. 4, 2009, ISSN 1842-3183
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MATHEMATICAL MODELLING WITH LAMB'S WAVES OF THE ULTRASONIC NON-DESTRUCTIVE CONTROL FOR THE MULTI-LAYER COMPOSITE MATERIALS

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Abstract: *This paper presents theoretical and experimental aspects related to the establishment of integrity status of the multi-layer composite materials. In order to do this ultrasounds will be used with relatively low frequency (100-500 kHz) in the shape of layer's waves (Lamb's waves). First, theoretical model of propagation of the Lamb's waves in a mono-layer isotropic material plaque situated in a fluid, and generalizing by using the Debye's series developing formalism it will determine the conception of a mathematical model adequate to the proposed purpose. The results obtained in such way were compared to those obtained through experiments, having as consequence that the mathematical model corresponds to the physical reality and can be successfully used in the determination of mechanical properties or of the integrity state's of a heterogeneous composite material.*

Key words: *ultrasounds, composites, Lamb's ways, acoustic field, Deby's series, non-destructive control.*

1. INTRODUCTION

The reflection and the transmission of the homogenous plane waves in an isotropic material layer with parallel faces is a subject of a lot of studies in ultrasonic field [1]. In the specialized literature we find works which treat the reflection and transmission of the heterogeneous waves through a solid simple interface and through a fluid layer. Much closer to the physical reality and to the proposed model of this article is the studies of [4] which deal with the case of an isotropic solid layer, viscous-elastic, immersed in a viscous fluid and therefore submitted to the activity of some of the heterogeneous plane waves. The structure of a composite material is mainly heterogeneous, because it is composed from a fiber texture more or less compact, included in a matrix, usually epoxy resin. From the specialized literature and taking into consideration the working frequencies situated somewhere in the interval between 100–500 kHz, it may be assimilated approximately and with a good reflection in the reality, all this structure as being homogenous. Instead of it, the presence of the fibers makes the composite material to have a strong anisotropic character.

2. PROBLEM FORMULATION

In order to calculate the dispersion curves of the Lamb's waves in a solid layer of an isotropic homogenous material, Viktorov [5] decomposes the acoustic field from the inside of the layer in the sum of the scalar potential Φ and the rotational potential vector $\vec{\Psi}$.

$$\vec{u} = \vec{\nabla}\Phi + \vec{\nabla} \wedge \vec{\Psi}. \tag{1}$$

The studded layer has infinite dimensions to directions 1 and 2 of the tri-axial orthogonal mark from the Fig. 1, and the thickness d, finite to direction 3.

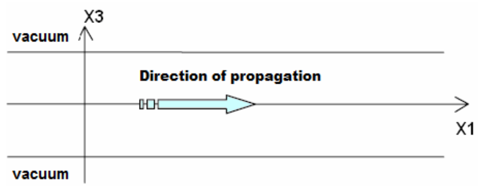

Fig. 1. The solid isotropic homogenous layer in the vacuum: references axis.

It will be considered as a plane non-homogenous wave propagating to the direction 1. The potentials are invariant translating to the direction 2; therefore all the physical values have partial derivates zero in relation with variable x_2. The scalar potential Φ and the vector potential $\vec{\Psi}$ will have the following form:

$$\Phi = \phi(x_3)e^{i(kx_1-\omega t)}, \tag{2}$$

$$\vec{\Psi} = \vec{\psi}(x_3)e^{i(kx_1-\omega t)}, \tag{3}$$

where:

- Φ the amplitude of scalar potential;
- $\vec{\Psi}$ the amplitude of the vector potential;
- k – wave number;
- ω – wave pulsation;
- t – time.

For a propagating wave with a wave number k and pulsation ω, the displacement vector component $\vec{u}$ will be:

$$u_1 = ik\phi - \frac{\partial\psi_2}{\partial x_3}, \tag{4}$$

$$u_3 = ik\psi_2 - \frac{\partial\phi}{\partial x_3}, \tag{5}$$

$$u_2 = -ik\psi_3 + \frac{\partial\psi_1}{\partial x_3}. \tag{6}$$

It can be noticed that equations (4) and (5) are coupled and depend only on the potentials Φ and Ψ_2. The equations describe the Lamb's wave that propagates in polarized plaque in sagital plan. The third equation is independent and describes the transversal wave polarized horizontally along the axis OX_2, named wave TH (transversal horizontal). In physical sense, the aim pursued was to model an incident wave with known characteristics which is propagate under a special angle in a composite material layer immersed in a fluid medium. The isotropic case: It will be treated further on the simplified case of the reflection and the refraction of a heterogeneous plane wave with unit amplitude at the origin, in an absorbent isotropic presumed layer, immersed in a fluid with knowing acoustic properties. The field of the acoustic displacement in the interior of the layer, in stationary regime, will be given by the formula:

$$\underline{U}(\underline{M},\omega,t)=\Re\left\{\sum_{p=1,2}\sum_{m=L,T}{}^{*}X_{pm}\,{}^{*}\underline{P}_{pm}\exp i(\omega t-{}^{*}\underline{K}_{pm}\underline{M})\right\}. \quad (7)$$

where:

- ${}^{*}X_{pm}$ the wave amplitude, the sign (*) denote the fact that the value may be complex;
- ${}^{*}\underline{P}_{pm}$ the wave polarization vector;
- ${}^{*}\underline{K}_{pm}$ wave vector;

$m = L, T$ the type of the propagated wave (L – longitudinal, T – transversal);

- $p = 1, 2$ the interface where the waves diffraction take place (1 – superior, 2 – inferior).

In the immersion fluid the field of the analogical acoustic displacement will be given by the relations:

$$\underline{U}_{1}(\underline{M},\omega,t)=\Re\left\{\begin{matrix}{}^{*}\underline{P}_{1}\exp i(\omega t-{}^{*}\underline{K}_{1}\underline{M})+{}^{*}X_{1R}\,{}^{*}\underline{P}_{1R}\\ \exp i(\omega t+{}^{*}\underline{K}_{1R}\underline{M})\end{matrix}\right\}. \quad (8)$$

For estimating the global reflection/refraction coefficients of the layer, i.e. for finding the amplitude expression of all the excited waves in a solid and fluid layer, it will be considered that each interface of the layer is an acoustic diopter between two semi-infinite mediums, situated on the distance of +/–d/2 on the median plan of it [3]. The problem will be reduced then to calculate three amplitudes corresponding to five elementary possible cases, considered separately. Each of this case it is a classical problem for four waves' propagation and the solution is discussed in specialized literature [2]. The obtained solutions should take into consideration the fact that the interface on which the waves' reflection/refraction occurred, it is situated at the distance +/-d/2 of the centered mark in the median plan of the layer. Thus, if ${}^{*}X^{0}_{ps}$ (p, s – conversion coefficient of the incidental type p wave in the type s transmitted/reflected wave) is the amplitude of the evaluated wave, placing the origin of the Cartesian mark on the interface, the expression of this amplitude on the distance z will be given with the relation:

$${}^{*}X_{ps}={}^{*}X^{0}_{ps}\exp(-i\,{}^{*}\phi_{m}), \quad (9)$$

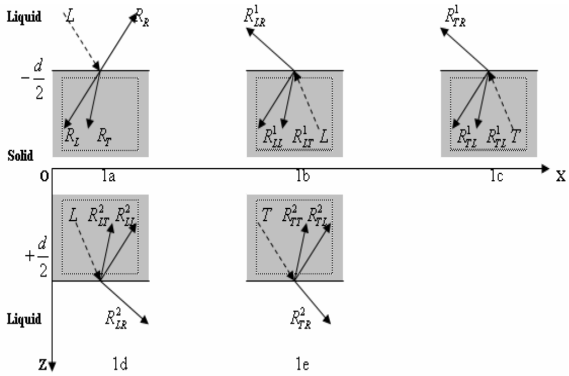

Fig. 2. Propagation in a semi-infinite medium; 5 possible cases.

where ${}^{*}\phi_{m}={}^{*}K_{m}z\cos{}^{*}\theta_{m}$.

The exponential $\exp(-i\,{}^{*}\phi_{m})$ it is called phase's factor, and ϕ_m it is called phase difference.

With a pointed line the incipient wave and with a continuous line the reflected and/or transmitted wave represented.

Further on, will be define three vectors composed from reflection and refraction coefficients of the layer, as follows:

$${}^{*}\underline{X}_{0}=\left\{{}^{*}R_{R},{}^{*}R_{L},{}^{*}R_{T}\right\}^{T}, \quad (10)$$

$${}^{*}\underline{X}_{p}=\left\{{}^{*}X_{pR},{}^{*}X_{pL},{}^{*}X_{pT}\right\}^{T}, \quad (11)$$

where the components ${}^{*}R_{m}\,(m=R,L,T)$, of the $\underline{X}_{0}$ vector from the 10-th relation, are respectively, R – reflection coefficient, L – longitudinal transmission and T – transversal transmission on the separation (dipoter) interface, fluid-solid, for an incidental wave in fluid.

In stead of it, vector's components ${}^{*}\underline{X}_{p}$, especially the quantities ${}^{*}X_{pm}$, with $(p=1,2)$ and (m = L, T) are relative to excited waves in solid, at the interface of the separation p.

For the case when is considered an incidental (pointed) wave which propagate in fluid in the sense of the Oz axis the components of the vector Xo will be obtained, Fig. 2,a.

For the rest of the cases, when the situation of some longitudinal and transversal waves incident on the 1 interface it is analyzed in turn, coming from the solid (the cases 2b and 2c), then of the incidence of the 2 surface of the same types of waves coming from the solid in the increasing sense of the Oz axis it will result the components of four reflection/refraction vectors on each interface. For each of the evaluated cases the amplitude of the incidental wave's it is considered known and equal with the unity.

In their turn, these one admit the definition of two matrixes of reflection/refraction on each interface, considered in the following manner:

$$[R_i]=\begin{bmatrix}0 & R^{i}_{LR} & R^{i}_{TR}\\ 0 & R^{i}_{LL} & R^{i}_{TL}\\ 0 & R^{i}_{LT} & R^{i}_{TT}\end{bmatrix},\ i=1,2. \quad (12)$$

Without relate in details of the calculus, the totality of the reflection/refraction coefficients presented in the above expression, is determined on writing the continuity equation in a point, on the considered interface [2].
It will be noted with $[T]$ and it is called double reflection matrix in the interior of the solid layer, the matrix:

$$[T]=[R_1][R_2]. \tag{13}$$

This one will have the form:

$$[T]=\begin{bmatrix} 0 & T_{LR} & T_{TR} \\ 0 & T_{LL} & T_{TL} \\ 0 & T_{LT} & T_{TT} \end{bmatrix}, \tag{14}$$

where the 6 non-zero coefficients of the double reflection matrix$[T]$, are given by the formula:

$$T_{ps}=R^1_{pL}R^2_{Ls}+R^1_{pT}R^2_{Ts}. \tag{15}$$

This double reflection matrix is the geometrics series ratio, named Debye's series.

$$\underline{X}_1^{(n)}=([1]+[T]+[T]^2+[T]^3+...+[T]^n)\underline{X}_0. \tag{16}$$

The coefficients' vector of the second interface it is obtained immediately with the relation:

$$\underline{X}_2^{(n)}=[R_2]\underline{X}_1^{(n)}. \tag{17}$$

The vector which gives reflection and transmission coefficients will be obtained taking into consideration an infinity of the successively reflections in the interior of the considered solid layer, i.e. it is the limit of the anterior series when (n) – is the number of the successively reflections, augments to the infinity.

We will have:

$$\underline{X}_1=([1]-[T])^{-1}\underline{X}_0, \tag{18}$$

$$\underline{X}_2=[R_2]\underline{X}_1. \tag{19}$$

It is important to emphasize that all of these calculated coefficients are related to propagation of the waves through a single acoustic diopter which separate the two mediums considered half-infinites, fluid-solid, though they define the propagation of the acoustic waves in the interior of the layer with two separated interfaces fluid-solid.

3 PROBLEM SOLUTION

For justification the use of the Debye's series, it will be evaluate in a transitory regime the response of an aluminum layer to the propagation of a homogenous plane wave.

In a classical mode, the evaluation of this response it is done by resolving a time convolution equation. The acoustical field resulted after scanning the solid layer it will be obtained with the Fourier integral in the frequency:

$$A(x,z,t,\theta)=\int_{-\infty}^{+\infty}E(f)H(f,\theta,z)\exp(i(2\pi ft))df, \tag{20}$$

where $E(f)$ is the Fourier's transformed data of the references sign and $H(f, \theta, z)$ is the function of transfer of the solid layer immersed in fluid (in this particular case the coefficient of transmission of the layer, i.e. X_{2R}, X_{2L}, and X_{2T}).

For simulating experimental conditions, the reference sign emitted by the palpate will be modulate being the multiplication between a function having a frequency f_0 and the Gauss's bell with the opening 2σ and the amplitude A_0.

The spectrum of such a signal will be (Fig. 3):

$$E(f)=A_0\exp(-\pi\tau^2[f-f_0]^2)\exp(i2\pi ft). \tag{21}$$

For the integral from the (20) relation, being unable to be analytically resolved, it was used a numeric method for its calculation, the continuous sum being replaced with a finite terms sum (the calculus being performed with a program written in C++ Builder5). In the following two figures are represented the transmitted sign by the aluminum layer for a incidental wave, calculated in two situations: when $H = X_{2R}$ taking into consideration all the internal reflections and when $H = X^{(1)}_{2R}$ is paid attention on a single doubled reflection in the interior of the layer.

The other parameters that interfere are:

It may be revealed that in the second case, that one using the Debye's series, the echo's become much more visible and easy to read in comparison with the first case, where the totality of the reflections determine an accentuated superposition of them.

$$k_{01}=8,\ k_{0L}=2,\ k_{0T}=4,\ \rho_1=1,\ \rho_{Al}=2.78,\ \frac{\omega d}{2\pi}=20,\ d=10\,\text{mm}.$$

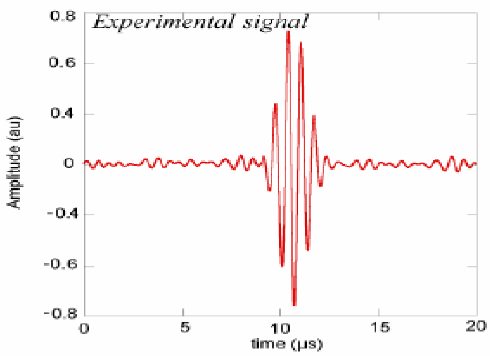

Fig. 3. The reference signal simulated depending on time.

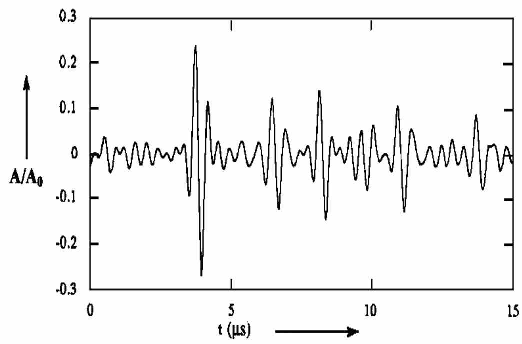

Fig. 4a. The response of the immersed layer using a classical transmitted coefficient.

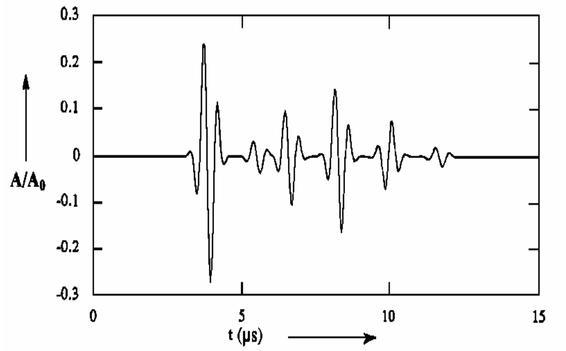

Fig. 4b. The response of the immersed layer using Debye's series.

The abundance of the visible signs in first case is caused by the discretisation of the spectrum, the calculated sign containing beside its useful part the superposition of all other trances with the same duration (20 μs) which constitute its "queue".

Instead of it, the use of Debye's series allow truncate the sign duration, fixing the repeated number of echoes in the interior of the layer, so that in correlation with the reverse Fourier's discrete transformed data, not to originate in undesirable superposition.

At the final, a confrontation theory-experiment done on a layer of composite material immersed in water confirm the anterior results. There are represented transmission and reflection modules depending on the angle of the incidence calculated theoretically with the (18) and (19) relations.

Several conclusions may be extracted: on the one hand the concordance of the theory with the experiment proves the existence of the heterogeneous plane waves' interaction in the interior of the layer immersed in the fluid. It is necessary to specify that all the anterior theory is not valid in case when the faces which form the diopter of the separation between the two mediums, fluid and solid, are not parallel.

The presented oscillations from the Figs. 5a and 5b are not caused by the numeric data processing but by the waves' interference phenomenon, which is though very amortized, keep being visible.

The other parameters which interfere in the calculus are:

$$k_{01} = 14.6,\ k_{0L} = 9.16,\ \alpha_{0L} = 0{,}083,\ k_{0T} = 1.7,$$

$$\alpha_{0T} = 0.25,\ \rho_1 = 1,\ \rho_{PVC} = 1.46,\ \frac{\omega d}{2\pi} = 35,$$

$$d = 10\ \text{mm},\ f = 3.5\ \text{MHz},\ C_{11} = 8.42,\ C_{44} = 1.82.$$

4. CONCLUSIONS

As a result of this study, it was demonstrated that the use of Debye's series in the calculus of reflection and transmission coefficients of a layer from a composite material, immersed in a fluid, it is justified from the physical reality point of view.

Definition of the mathematical machine, as well as its inclusion in a specialized software application on a platform C++ Builder5, it will permit in the future the imagination and prosecution of a numerous trying for determination of the behavior of different materials while passing ultrasound waves.

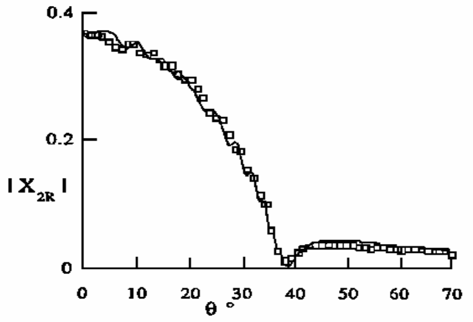

Fig. 5a. The global transmitting coefficient module depending on the incidental angle continuous lines – theory, square – experiment.

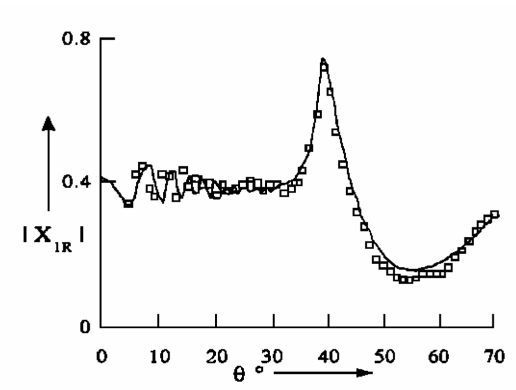

Fig. 5b. The global reflection coefficient module depending on the incidental angle continuous lines – theory, square – experiment.

The particular application for which was created the program and especially that to be use for the study of composite materials, may be easily extended and to other situations excepting the fluid immersion, because this software contains modules easy to be change and use for any other application from the domain.

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