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NEW STOCHASTIC MODELS APPLIED TO MANUFACTURING SYSTEMS

Adrian Stere PARIS[1], Constantin TÂRCOLEA[2]

Abstract: *A classical method of the analyzing the lifetime of the manufacturing systems is to use distributional laws, which take values on the bounded intervals, within which the stochastic repartitions are constrained to vary. Some adequate bounded distributions derived from classical models are developed and compared in the paper. The proposed laws are used for modeling reliability function and hazard-rate. The numerical examples prove that these models are adequate for practical applications.*

Key words: *lifetime distributions, failure function, hazard-rate function.*

Motto:"So far as theories of mathematics are about reality, they are not certain, they are not about reality."
Albert Einstein

1. INTRODUCTION AND LITERATURE REVIEW

The term *Manufacturing* refers to a sequence of steps, typically carried out by multiple departments, that take various inputs, (such as raw materials, subassemblies) and covert them into desired outputs (other assemblies or finished products using limited resources (such as machines, personnel, computers). Each step involves ones or more on the following: a *conversion activity* that adds value to the product; a *material-flow activity* that moves material from one location to another or an *information-flow activity* that helps in coordination of the material flow and conversion activities and also provides feedback for continuous improvement in the factories products, procedures and operating characteristics. Most manufacturing processes are subject to random disruptions caused by machine failures, arrival of demands, changes in product design etc. [12].

In the new economy, business requires their manufacturing systems to maintain low inventory levels and provide short delivery lead times. Machining systems are therefore being called upon to reliably manufacture products at varying rates dependent on the products instantaneous demand [7]. Furthermore, decreasing product life cycle lengths have reduced the time available to develop machining systems. As a result, machining system designers need analytical models to predict system performance under various production scenarios [3].

The first paper that discusses the evolution of manufacturing systems based on natural variability and randomness was Wright's study [19] on machine interference problem, published in 1936. Many stochastic models have been proposed and published [5, 6, 7, 10, and 14]. The theoretical results of the studies should be used to gain understanding and anticipating the possible causes of failures during the lifetime and prognoses of a given system [11].

Today the reliability of engineering systems has become an important factor during their planning design, and operation. The factors that are responsible for this include high acquisition cost, increasing number of reliability-related lawsuits, complex and sophisticated systems, competition, public pressures, and the past well-publicized system failures. Needless to say, over the past 60 years, many new advances have been made in the field of reliability that help to produce reliable systems [6].

The reliability and safety analysis and assessment of complex systems is becoming more and more difficult task due to the fact that the reliability and safety of manufacturing systems depend not only on the failed states of system components, but also on the sequence of occurrences of those functions [1].

Linear models are used for a good representation of reality or are an approximation to a reality [8]. Often for describing the physical and engineering phenomena it is necessary and possible the using nonlinear model too [4, 15, 16, and 17].

Regression is a simple technique for investigating relationships between output and input variables of a manufacturing process [3, 11, and 18]. Different nonlinear stochastically distributions are presented, because the statistical laws are a measure of expressing of uncertainty about the randomness of the processes and the work conditions vary over time.

This study proposes and compares a few stochastic distributions capable of modeling the probabilistic concepts of survivorship (reliability function), and hazard-rate.

A main goal of the paper was to find adaptive curves for the reliability theory. The classical laws can be adapted for describing data from lifetimes and fatigue, because the stochastic models could be fitted to the experimental data [2, 10, 12, and 13]. Regression techniques provided tools for analysis of failures, using failure function, hazard-rate, and predictions. The estimat-

[1] PhD, Math., Professor, University "Politehnica" of Bucharest, Department of Mathematics, Splaiul Independentei 313, Sector 6, 060042, Bucharest, ,Romania,
E-mail: constantin_tarcolea@yahoo.com
[2] PhD, Eng, Lecturer, University "Politehnica" of Bucharest, Materials Tehnology and Welding Department, Splaiul Independentei 313, Sector 6, 060042, Bucharest, Romania,
E-mail: adrianparis@amza.camis.pub.ro

ing of the parameters of regressions models were made with the software CurveExpert 1.4.

The regression models have a good fitting to experimental data as shown by the s and r coefficients.

2. METHODS AND PROCEDURES

The reliability function is a corner-stone of reliability theory. Denoting the lifetime variable of the system by T the reliability function, $R(t)$, is the probability that the system will survive to time t:

$$R(t) = P(T > t)\,. \tag{1}$$

The failure (cumulative distribution) function, F(t), it is defined by:

$$F(t) = 1 - R(t)\,. \tag{2}$$

It results, that is a one-to-one correspondence between $R(t)$ and $F(t)$. Therefore $R(t)$ contains no new information's beyond what is contained in $R(t)$. In other words the failure function of the discrete or continuous random variable is the mapping with definition domain R_+, whose values are in [0;1], given by equality:

$$F(t) = P(T \le t)\,. \tag{3}$$

In reliability researches the most frequently applications track the exponential distribution and the more generally the classical Weibull distribution.

In this study we analyzed the following continuous failure functions:

- Hoerl model failure function

$$y(x) = ab^x x^c\,; \tag{4}$$

- modified exponential failure function

$$y(x) = ae^{\frac{b}{x}}\,; \tag{5}$$

- sinus model failure function

$$y(x) = a + b\cos(cx + d)\,; \tag{6}$$

- vapor pressure model failure function

$$y(x) = e^{a + \frac{b}{x} + c\ln x}\,. \tag{7}$$

The experimental data and the regression curves of the proposed models (eq. 4, 5, 6, 7) were graphically represented in the Figs. 1, 2, 3, 4:

The lifetime's laws purport the distribution functions of a duration variable. The concept known as hazard-rate is the instantaneous failure rate and plays an important role in the reliability theory and practice. The hazard-function can be introduced in the following way: first it is defined the hazard on the interval $(t;\, t + \Delta t]$.

Hazard $(t;\, t + \Delta t]$= $P(t < T \le t + \Delta t\,/\,T > t) =$

Probability that the system will fail in the time period t to $t + \Delta t$, given that it has lasted until time t.

Based on conditional probabilities and definitions of reliability function it results:

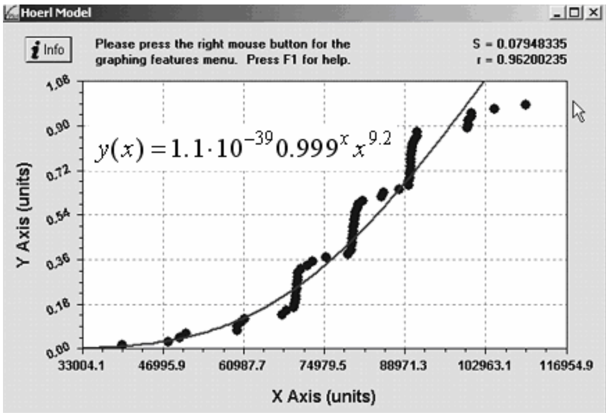

Fig. 1. Hoerl failure function.

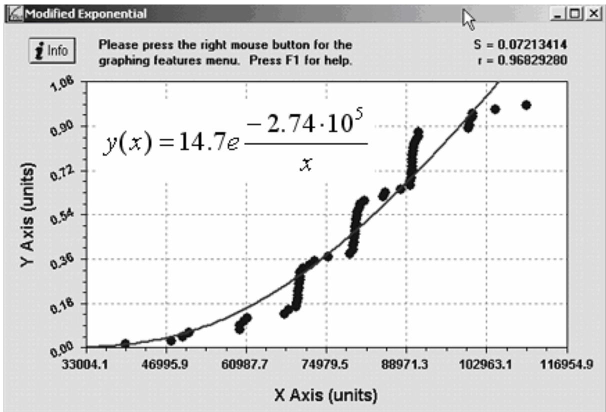

Fig. 2. Modified exponential failure function.

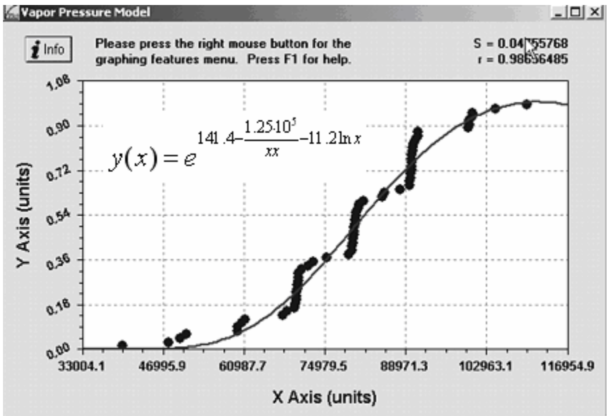

Fig. 3. Vapor pressure failure function.

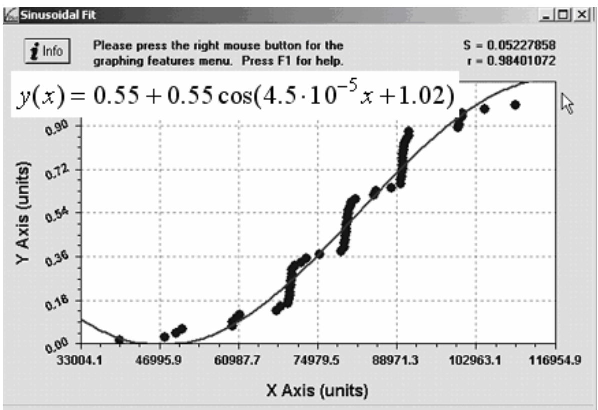

Fig. 4. Sinus model failure function.

$$\text{Hazard } (t;\, t+\Delta t] \cong \frac{-R^{'}(t)}{R(t)}\Delta t \;\text{ if }\; \Delta t \;\text{ is small enough.}$$

Taking the limit, it is obtained so called hazard-rate function:

$$(t) = \frac{-R^{'}(t)}{R(t)}. \tag{8}$$

There is a one-to-one correspondence between $R(t)$ and $r(t)$ too. In other words if $R(t)$ is known is once $r(t)$ is known:

$$R(t) = e^{-\int_0^t r(u)du}. \tag{9}$$

The bath-tube curve does not depict the failure-rate of a single item; it depicts the relative failure-rate of entire population of items over time.

In the present paper we used the following stochastic laws as the hazard-rate functions:

- Hoerl model hazard-rate function;
- Sinus model hazard-rate function;
- Polynomial hazard-rate function;
- Reciprocal quadratic hazard-rate function.

For a experimental data set and the above functions the regression curves were graphically represented in the Figs 5, 6, 7, and 8.

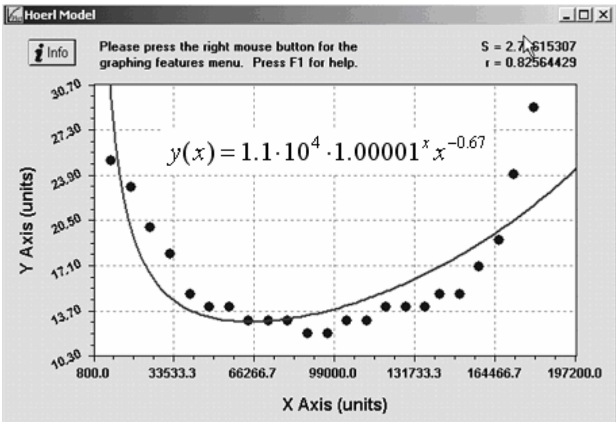

Fig. 5. Hoerl hazard-rate function.

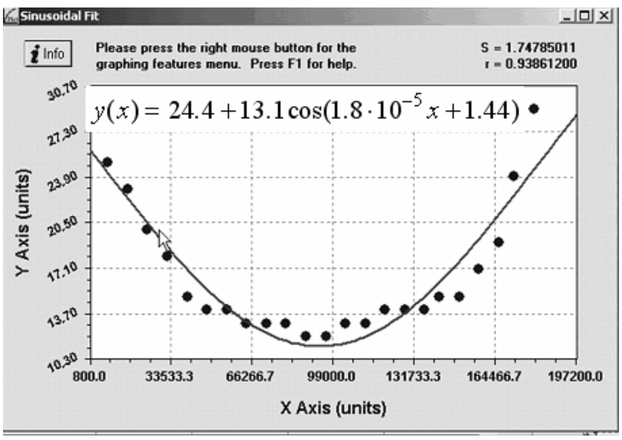

Fig. 6. Sinus hazard-rate function.

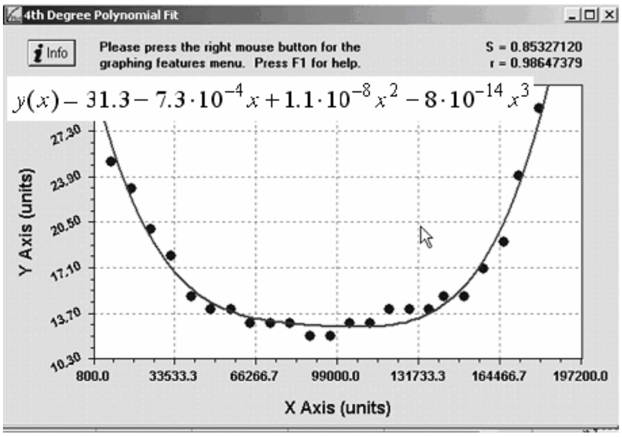

Fig. 7. Polynomial hazard-rate function:

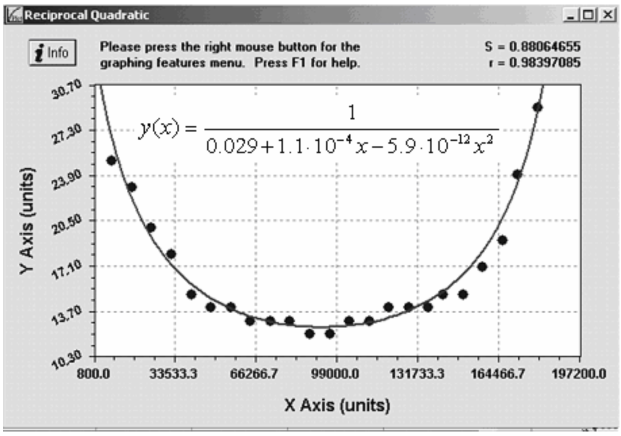

Fig. 8. Reciprocal quadratic hazard-rate function.

3. CONCLUSIONS

Statistical process control contains statistical techniques used for improvement of manufacturing processes. In the paper it was proposed adaptive stochastic distributions for describing the lifetime of the usual manufacturing models. The study utility consists in the fact that the obtained results can be useful in the design of the actual strategies and in the practice allowing the prediction of failures for the manufacturing systems. In the first part of the paper were introduced adequate bounded distributions for estimation the evolution of the cumulative distribution function. It was used the regression technique, which has the property that the sum of squared vertical distances from it to point on the scatter diagram is a minimum. The best correlation coefficient, 0.986, has the vapor pressure model.

Given a lifetime data set we applied a few rata-failure models. From the analysis of residual sum of square and correlation coefficients it results that the proposed laws have the graphs closed to experimental data. It can see that the best correlation coefficient, 0.986, has the polynomial model. The introduced distributions allow a good describing of the failure function respectively hazard rate, taking the form of the increasing theoretical cumulative distribution function respectively the form of the classic curve of bath tube, both useful for modeling of practical cases.

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