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# MINIMIZING THE ERRORS OF SURFACE MACHINING THROUGH INTERPOLATION ON NUMERICAL CONTROL MACHINE TOOLS

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***Abstract:*** *The main theme refers to the increase of the machining accuracy through minimizing the path error while machining through circular, or other kind of interpolation on numerical control milling, boring and milling grinding or turning machines. Currently the process of path error minimization while machining on numerical control machine tools is explored at a low extent because of the absence of the dependence relations between the path error and the parameters of the transient duty of each kinematical axis taking part to the interpolation of the work piece contour. This work is also establishing and analyzing the relations of dependence between the path error and the response time of the kinematical axes involved in machining through circular interpolation.*

***Key words:*** *contour, error, interpolation, response time, machine tools.*

## 1. INTRODUCTION

The numerical control machine tools are able to machine various shapes of work pieces where different types of interpolations (linear, circular, helical, sinusoidal, etc.) are applied. The contour resulting after machining will have a certain error compared to the programmed contour. The error of the work piece contour has three major components [1, 3]:

- error caused by the machine geometry;
- error caused by the rigidity of the machine-work piece-tool system;
- error caused by the transient duty of the kinematical axes taking part to machining the contour.

The most prominent is the last component, known as the path error. This will be analyzed and researched further on. Machining a contour through interpolation involves establishing a large number of points on the work piece contour that will be concretized through eth correlation of the motion of the kinematical axes of the machine. The feed rates of the kinematical axes taking part to interpolation have to be correlated so that [4]:

- the resultant of the speeds should permanently remain tangent to the requested contour;
- the size of the speed resultant should be constant at any point of the contour in order to obtain the same quality of the surface being machined.

The block of the numerical control equipment designed to carry out this task, denominated interpolator, receives a minimal number of data needed for a complete and univocal defining of the contour to be run, for instance: the coordinates of the initial point and of the final point, characteristics of the path to be run between the initial and final points, value of the feed rate. Based on these data the interpolator will calculate the required contour that is to be run at a constant speed.

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## 2. THE RELATION OF DEPENDENCE BETWEEN THE PATH ERROR AND THE PARAMETERS OF THE KINEMATICAL LINKAGES AT CIRCULAR INTERPOLATION

A numerical control milling machine being composed of three kinematical axes *X*, *Y* and *Z* is considered. In order to establish the relation of dependence between the path error and the parameters of the kinematical linkages, a surface is machined through circular interpolation to which the *X* and *Y* axes are taking part. Running on contours described by a circle is a method frequently used on CNC machine tools. The circle segment *OAB* is considered to be machined through circular interpolation at the speed $V$ obtained through motions along the axes *OX* and *OY* at the feed rates $V_x$ and $V_y$. *A* is a point, whatever, on this path, Fig. 1. Because of the delay on axes caused by the difference between the command moment and its performance moment, a time interval $t_0$, comes up that makes the actual path of the tool be different from the programmed one. For bringing this case of the circular interpolation to eth case of linear interpolation in terms of calculation procedure, the circle segment *OAB* will be considered deployed.

Thus, the length of the programmed *OAB* circle segment is equal to the length of the segment *O'B'* having the value $2\pi R / \gamma$, where the symbols mean:

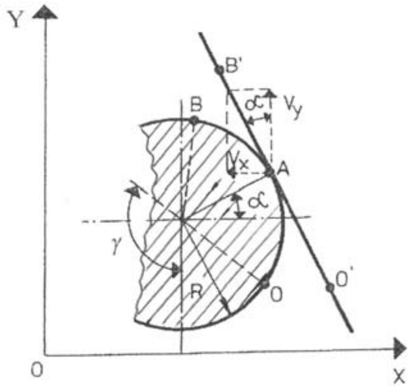


**Fig. 1.** Path error at circular interpolation.

- $R$ – the radius of the interpolated circle;
- $\gamma$ – the angle of the circle arc $OAB$.

The speed rate $V_x$ and $V_y$ on the two controlled axes have the following values while machining the circle arc [2, 5]:

$$\begin{aligned} V_x &= -V\cos\alpha \\ V_y &= V\sin\alpha \end{aligned}, \tag{1}$$

where $\alpha$ is the angle towards the center of the interpolated circle. By considering that there is an average value for $V_x$ and $V_y$ then the case of the circular interpolation is reduced to achieving the interpolation of the linear segment $O'B'$, with the mention that in the relation of the path error, the feed rates of the two kinematical linkages will be the ones from the circular interpolation. As such, the rates of the follow-up error on the two axes will be:

$$\begin{aligned} \xi_x &= V_x^{'} t_{rx} - \frac{a_x t_{rx}^2}{2}, \\ \xi_y &= V_y^{'} t_{ry} - \frac{a_y t_{ry}^2}{2}. \end{aligned} \tag{2}$$

where: $V_x'$ and $V_y'$ mean the average values of the speeds $V_x$ and $V_y$ while running along the segment $O'AB'$; $t_{rx}$ and $t_{ry}$ mean the response times of the $X$ and $Y$ axes; $a_x$ and $a_y$ mean the accelerations of the kinematical linkages on the $X$ and $Y$ axes.

$$\frac{a_x t_{rx}^2}{2} = \frac{V_x^{'} t_{rx}}{2}\,;\ \frac{a_y t_{ry}^2}{2} = \frac{V_y^{'} t_{ry}}{2}. \tag{3}$$

After replacing the relations (3) into (2) it will result:

$$\xi_x = \frac{V_x^{'}}{2} t_{rx}\,;\ \xi_y = \frac{V_y^{'}}{2} t_{ry}. \tag{4}$$

The values of the follow-up errors along the circle arc $OAB$ will be:

$$\begin{aligned} \xi_x &= -\frac{V\sin\alpha}{2} t_{rx}; \\ \xi_y &= \frac{V\cos\alpha}{2} t_{ry}. \end{aligned} \tag{5}$$

By replacing the relations (1) and (5) into the relation of the path error $e$ at linear interpolation [3] it will result:

$$e = \frac{V^2}{2}\cos\alpha\,\sin\alpha\,(\frac{\cos\alpha}{a_y} + \frac{\sin\alpha}{a_x}). \tag{6}$$

The relation (6) expresses the relation between the path error $e$ and the accelerations of the two kinematical linkages. In function of the quadrant where the circle arc is located, the value of the two speeds will be either plus or minus and the value of the path error will be plus or minus, as well. The analysis of the relation (6) in case when the circle arc would become a circle, leads to the conclusion that for $\alpha$ = 0º; 90º; 180º; 270º the value of the path error is zero. The maximum value of the error will be met for $\alpha$ = 45º; 135º; 225º; 315º. By analyzing the term:

$$\frac{\cos\alpha}{a_y} + \frac{\sin\alpha}{a_x}, \tag{7}$$

it may be noticed that at the points α = $K \times 45$º, where $K$ = 1…4, the relation (8) becomes:

$$\frac{\sqrt{2}}{2}(\frac{a_x + a_y}{a_x a_y}). \tag{8}$$

If $a_x \neq a_y$, the value of the term (8) will be higher than in case of $a_x = a_y$. If $a_x = a_y = a$, the value of the term (8) will be $\sqrt{2}/a$. The more the acceleration of the two kinematical linkages is higher the value of the path error will be lower. In the process of establishing the transient duty that is done even from the machine design stage, the accelerations on the machine axes should be equal for axes taking part to interpolations and the acceleration rates should at the same time be as high as possible. When the acceleration rate of the two kinematical linkages differs, the circle obtained through circular interpolation will have deviations from roundness with maximums at the points α = $K \times 45$º, where $K$ = 1…4. The more the difference between the accelerations $a_x$ and $a_y$ is higher, the roundness deviation will increase.

## 3. EXPERIMENTAL TRIALS

The experimental trials are consisting of machining (milling) a circular surface through interpolation, having a diameter within 230–250 mm. The work piece is made of cast iron of 200–210 HB and the roughness of the machined surface is $R_a$ = 3.2 μm. The tool being used for milling is a cylindrical frontal mill cutter with metal carbide inserts 8 teeth and 63 mm diameter. Because the path error is also including the feed rate $V$ at which the contour is run, cutting tests will be done, through keeping constant the other parameters and modifying the feed rate. One circle is to be machined through interpolation under each one of the following conditions:

- the kinematical axes $X$ and $Y$ have equal transient times and feed rate $V$ = 140 mm/min;
- kinematical axes $X$ and $Y$ have different transient times and feed rate $V$ = 140 mm/min;
- kinematical axes $X$ and $Y$ have different transient times and feed rate $V$ = 180 mm/min.

a) Experimental trials when the kinematical axes $X$ and $Y$ have equal transient times and feed rate $V$ =140 mm/min. The transient duties on the X and Y axes are:

$$X\text{ axis: } \begin{cases} a_x = 1.1 \\ K_v = 1.01 \end{cases};\ Y\text{ axis: } \begin{cases} a_y = 1.1 \\ K_v = 1.01 \end{cases}. \tag{9}$$

where $K_v$ is the amplification factor and $a$ is the acceleration of the axis, established through the machine data. The profile diagram of the circle being machined is shown in Fig. 2.

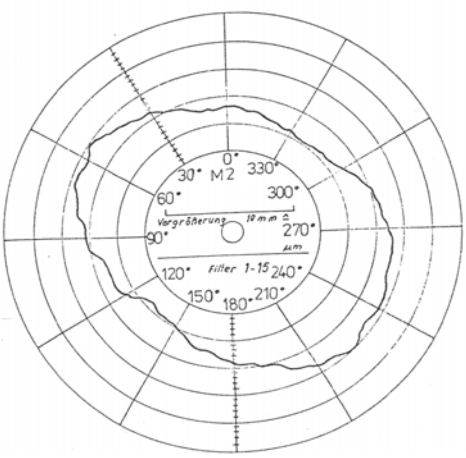


**Fig. 2.** Profile diagram in case of equal transient duties ($V$ = 140 mm/min), $e_{\max} = 30.4$ μm.

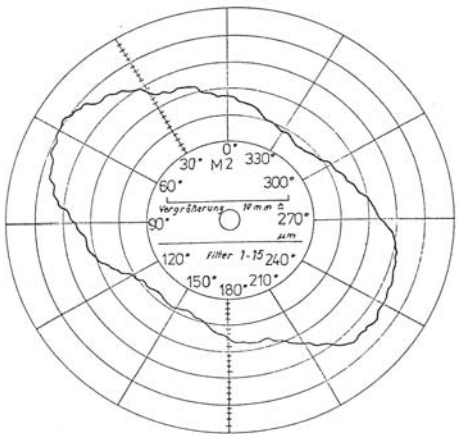


**Fig. 3.** Profile diagrams in case of different transient duties ($V$ = 140 mm/min), $e_{\max} = 40.7$ μm.

b) Experimental trials when the kinematical axes $X$ and $Y$ have different transient times and feed rate $V$ = 140 mm/min. The transient duties on the $X$ and $Y$ axes are:

$$X\,\text{axis:}\ \begin{cases} a_x = 1.1 \\ K_v = 1.01 \end{cases};\ Y\,\text{axis:}\ \begin{cases} a_y = 0.75 \\ K_v = 1.01 \end{cases}. \tag{10}$$

The profile diagram of the circle being machined is shown in Fig. 3.

c) Experimental trials when the kinematical axes X and Y have different transient times and feed rate $V$ = 180 mm/min. The transient duties on the X and Y axes are:

$$X\,\text{axis:}\ \begin{cases} a_x = 1.1 \\ K_v = 1.01 \end{cases};\ Y\,\text{axis:}\ \begin{cases} a_y = 0.75 \\ K_v = 1.01 \end{cases}. \tag{11}$$

The profile diagram of the circle being machined is shown in Fig. 4.

d) Experimental trials when the kinematical axes $X$ and $Y$ have equal transient times and feed rate $V$ = 140 mm/min. The transient duties on the $X$ and $Y$ axes are:

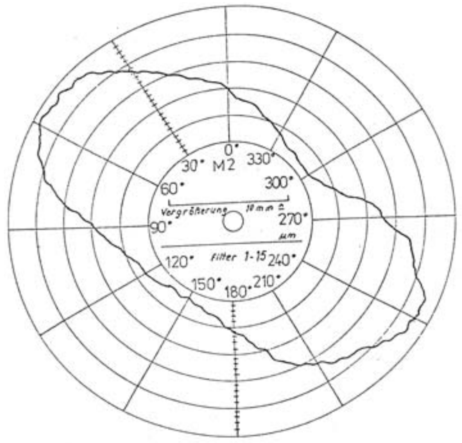


**Fig. 4.** Profile diagrams in case of different transient duties ($V$ = 180 mm/min), $e_{\max} = 50.2$ μm.

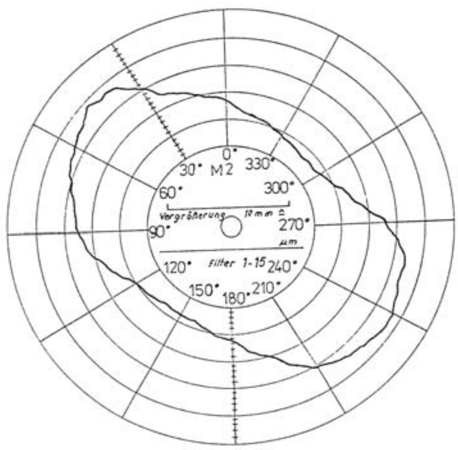


**Fig. 5.** Profile diagram in case of equal transient duties ($V$= 140 mm/min), $e_{\max} = 39.6$ μm.

$$\text{X axis:}\ \begin{cases} a_x = 0.75 \\ K_v = 1.01 \end{cases};\ \text{Y axis:}\ \begin{cases} a = 0.75 \\ K_v = 1.01 \end{cases}. \tag{12}$$

The profile diagram of the circle being machined is shown in Fig. 5.

e) Experimental trials when the kinematical axes $X$ and $Y$ have different transient times and feed rate $V$ = 140 mm/min. The transient duties on the $X$ and $Y$ axes are:

$$X\,\text{axis:}\ \begin{cases} a_x = 0.75 \\ K_v = 1.01 \end{cases};\ Y\,\text{axis:}\ \begin{cases} a_y = 0.5 \\ K_v = 1.01 \end{cases}. \tag{13}$$

The profile diagram of the circle being machined is shown in Fig. 6.

f) Experimental trials when the kinematical axes $X$ and $Y$ have different transient times and feed rate $V$ = 180 mm/min. The transient duties on the $X$ and $Y$ axes are:

$$X\,\text{axis:}\ \begin{cases} a_x = 0.75 \\ K_v = 1.01 \end{cases};\ Y\,\text{axis:}\ \begin{cases} a_y = 0.5 \\ K_v = 1.01 \end{cases}. \tag{14}$$

The profile diagram of the circle being machined is shown in Fig. 7. Through the analysis of the profile diagrams it may be noticed:

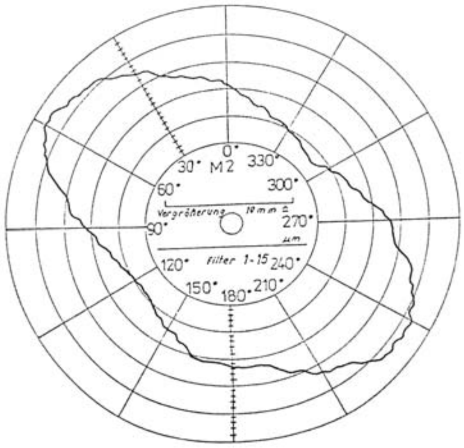


**Fig. 6.** Profile diagrams in case of different transient duties ($V$ = 140 mm/min), $e_{\max} = 46.1$ μm.

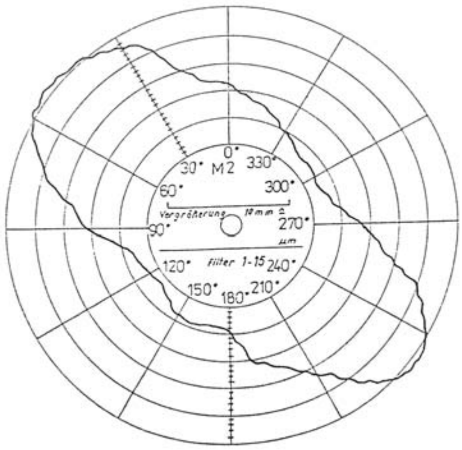


**Fig. 7.** Profile diagrams in case of different transient duties ($V$ = 180 mm/min), $e_{\max} = 55.2$ μm.

- In case of the experiment where the transient duties of the two axes are equal, Fig. 2 and Fig. 5, the maximum deviation from roundness will be at the angles α = 45º; 135º; 225º; 315º.
- In case of the experiment where the transient duties of the two axes are different, an increase of the roundness deviation has resulted compared to the first case, Fig. 3 and Fig. 6, having the maximum deviation from roundness at the same points where α = 45º; 135º; 225º; 315º.
- The increase of the circle running speed rate has led to the increase of the roundness deviation, Fig. 4 and Fig. 7.

This analysis of the experimental results confirms the validity of the dependence relation between the path error and the transient duties of the axes taking part to interpolation.

## 4. CONCLUSIONS

Based on the relations of dependence between the path error and response time of the kinematical axes taking part to interpolation, conclusions may be made, that are useful for both the machine programmer and the numerical control machine tool designer. The programmer may use the relations of the path error and the experimental results for assessing the capability of the machine tool with numerical control in terms of the machining accuracy obtained through interpolation. At the same time, based on the relations being established, the programmer can modify several parameters related to the cutting duty (especially the feed rate) and to the set-up of the work piece on the machine table in order to equalize the response times on axes, so that the rate of the path error is lowered up to an admissible value, imposed by the work piece requirements. The programmer may also intervene with likely contour geometrical corrections at the stage of part programming. The designer of the numerical control machine tool, by having the relations of the path error, has the possibility to begin the design activity from the value of the path error and of the cutting duty, for establishing the response time of the axes. With these response times the acceleration/deceleration rates of the kinematical axes can be determined. Through the analysis of the relations of dependence, the machine designer has to achieve a equalization of the response times for the axes taking part to interpolation, with a view to lowering the path error. The value of the response time has to be as low as possible. This thing requires, even from the design stage, the optimization of the kinematical linkage sizing. For several machines whose structure includes kinematical axes of high inertia (such as column motions) it has to be considered these axes not to take part to interpolation ($Z$ axis). Several numerical control machines whose axes often participating to interpolation ($X$ and $Y$ axes) have different inertia rates it obvious that will not be recommended, as the path error will be high.

This manner of establishing the relations of dependence can also be extended to the other types of interpolation: evolving, helical, sinusoidal, hyperbolical etc. If the response form of the transient duty is different from linear, the algorithm for establishing the path error will be same up to the moment of establishing the follow-up errors on the kinematical axes.

## REFERENCES

[1] Y. Bell, H. South, *Compensation system of backlash and pitch errors of the feed kinematical linkages of the numerical control machine tools,* Progresivie Tehnologii i Sistemi Masinoctroenia, 2005, pp. 165–168, Donet.

[2] P. Bezier, A. Pall, *Optimizing the transmission ratio on mechanical systems for decreasing the response time,* Buletinul Universitații "Gh. Asachi" din Iaşi, Tomul XLVII(LI), 2004, pp. 555–558.

[3] G. Stan, *Optimizing the choice of the response times of the industrial robot servomechanisms in order to minimize the path error*, ModTeh International Conference, 2010, pp. 551–554, Iasi.

[4] V. Ungureanu, *Positioning accuracy at direct and indirect measurement of robot kinematical linkages*, Buletinul Universitații "Gh. Asachi" din Iași, Tomul XLVI, 2005, pp. 251–256.

[5] V. Ungureanu, *Improving measurements for trancitional regimes of feed kinematic chain from machine-tools with numerical control*, International DAAAM Symposium "Inteligent Manufacturing & automation", 2008, pp. 555–558, Viena.