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CRACK DEVELOPMENT IN LINER USING CAGINALP PHASE FIELD MODEL

Marius TURNEA[1], Dragoş AROTĂRIŢEI[2,*], Mihai ILEA[3], George CONSTANTIN[4], Mariana ROTARIU[5]

[1)] Lecturer, PhD, Dept. Medical Bioengineering, University of Medicine and Pharmacy "Grigore T. Popa", Iasi, Romania
[2)] Prof., PhD, Dept. Medical Bioengineering, Univ. of Medicine and Pharmacy "Grigore T. Popa", Iasi, Romania
[3)] Lecturer, PhD, Dept. Medical Bioengineering, Univ. of Medicine and Pharmacy "Grigore T. Popa", Iasi, Romania
[4)] Prof., PhD, Machine and Manufacturing Systems Department, University "Politehnica" of Bucharest, Romania
[5)] Lecturer, PhD, Dept. Medical Bioengineering, Univ. of Medicine and Pharmacy "Grigore T. Popa", Iasi, Romania

Abstract: *The interface between stump and liner (socket) is subject of high pressure in different moments of gait cycle due to loading variation in body movement. The liner is the interface between socket and skin along with soft tissue and bones in a cycling loading. The result is a friction effect combined with pressure that conduct to the heating of skin and liner. The effect is and increasing of pressure and friction and the increasing the temperature again, in a cascading cycle than can conducts to fracture of liner and scars on stump.*

There are models that describe the fracture development and prediction of crack propagation but in the last years, an increased attention is given to phase field models applied to fracture development as a better solution for analysis of fracture in different materials of composites. A simplified approach that uses Caginalp model with memory is used to predict the fracture of liner under cycling loading. The numerical results of simulations are presented as outcomes of the theoretical approach.

Key words: *Caginalp model, phase fields models, finite element method, crack development, fracture analysis, numerical methods, finite difference method.*

1. INTRODUCTION

The main objective of prosthetic by patient’s point of view is related to comfort [1]. A correct measure of the patient’s satisfaction is difficult to be made because usually, the degree of satisfaction is measured using different scales, but all of them are based on subjective response to more or less subjective questions from standardized questionnaires [1].

The pressure at tegument level is a major part of discomfort and as follow, medical problems: scars, inflammation and so one. The interface at level of blunt-liner has a distinctive achievement in a comfortable prosthesis for a person with the level lower limb amputation. Amputation is a surgery that involves the section of anatomic parts: bones, muscles, blood vessels and nerves. After the surgery, the tissues are sensitive and will bear heavy pressure and shear stresses induced by prosthesis. In order to avoid these problems, two basic solutions are taken into account: adjustment of the actually prosthesis during wear and construction of some improvements that relief the pressure in the affected zone, e.g. compliant prosthetic sockets [2].

Finite element method (FEM) is used in many papers that deal with prosthetic aspect and analysis of stress in soft tissues, bones, interface levels or gait cycles [2−4]. A FEM model is used for analysis of patellar tendon, in condition of quasi-static loading in condition of normal loading. The compliances are used to relief the pressure in fibula head [2].

A study about the internal mechanical condition in soft tissue is done in [3]. Transtibial amputee is the subject of FEM simulation and the displacements of residual tibia and fibula from simulations are validated by MRI measurements. The internal strain, stress and strain energy density are measured in most critical points, under truncated bones [3].

Very few articles refer to aspects of tribology interface between skin and liner in a complex that use the stump-liner socket assembly [4]. In the known cases, all the research aspects refer only to experimental results and simulation using FEM in order to describe a map of pressure and stress over the stump surface [4].

The analysis of pressure and friction at interface level between skin and liner or between skin and socket (cup) is one of the greatest for patient's health conditions. An inadequate contact interface can produce affections of teguments like scars that are particularly difficult cases for treatment. Wear and friction for healthy and amputated member is studied in [5]. The authors used a micro-tribometer in order to obtain experimental results.

The usually approach for skin in FEM modeling is homogenous and isotropic material. Some of the approaches use a hyperelastic model (e.g. Mooney-Rivlin). The elasticity modulus of skin has the major influence in model with FEM for assembly soft-tissue/skin/liner or soft-tissue/skin/socket [6].

* Corresponding author: str. Kogalniceanu, nr. 9-13, Iasi, Romania, Tel.: +40.232.213.573
E-mail addresses: *dragos_aro@yahoo.com* (D. Arotaritei)

Fracture in materials can happen in various forms in different medical applications. In recent years, the introduction of phase field model that describes the kinetic of transition between two phases (in our case two different materials) was proven to be useful for crack modeling. A parameter makes distinction between two phases of the material, the solid one and the "broken" one, the crack.

Sharp fracture presents discontinuities that create difficulties in computational model for crack topologies. Based on crack surface density function, the authors proposed to use a function that describes at macroscopic level the crack surface in material [7].

ABAQUS is a suitable CAD software to develop own model for fractures. UEL and UMAT subroutines can be used to develop phase field model for brittle fracture numerical approach [8]. A regularization parameter controls the diffusion process and diffusion equation is solved numerically by Newton–Raphson method [8]. A 2D phase-field model with no anisotropy is proposed in [9] using a numerical solution. The assertions of authors are proved numerically in an application case, the super cooled solidification [9].

FEM is used in modeling the crack pattern in a single or composite material [10]. The crack pattern is described by the field variable. The coupled equations that are used in this approach are based on Ginzburg-Landau (and Allen-Cahn) phase field equations from phase field theory and elasticity equations [10].

An interesting approach is proposed in [11]. The authors proposed a phase-field model for cohesive fracture and Dirac function is used to do the phase-field approximations [12].

In what follows we propose an originally approach, an phase field model, Caginalp based approach that can be used for materials with memory in conjunction with temperature evolution due to frictional caused at the phase field interface.

2. CAGINALP PHASE FIELD MODEL APPLIED TO CRACK DEVELOPMENT

Based on Landau–Ginzburg approach, Caginalp [11−14] proposed a phase-field model that incorporated surface tension, anisotropy, curvature and dynamics of the interface (Fig. 1).

Phase field is practically in our case a mathematical tool that converts a moving boundary problem into a set of partial differential equations, which can be solved numerically.

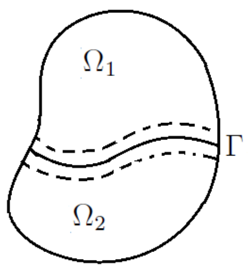

Fig. 1. Material that occupies a zone Ω can exists in two phases: Ω_1 – liquid or Ω_2 – solid. The phases are separated by an interface Γ.

The dotted lines in Fig. 1 indicate a possible interface between two phases.

An analytical solution is very difficult to find, in many engineering approaches as biomedical engineering, numerical values over a definition domain are acceptable solution.

We consider the problem of initial values and bounds that is the Caginalp model of phase transitions for materials with memory:

$$\begin{cases} \varphi_t(t,x) - \dfrac{\xi^2}{\tau}\Delta\varphi(t,x) + \dfrac{1}{2\tau}(\varphi^3 - \varphi)(t,x) = \\ = \dfrac{2}{\tau}u(t,x) + f(x,t), (t,x) \in R_+ \times \Omega \\ (u(t,x) + \dfrac{l}{2}\varphi(t,x))_t - \displaystyle\int_{-\infty}^{t} a(t-s)\Delta u(s,x)dx = g(t,x) \end{cases}. \quad (1)$$

$$\begin{cases} \varphi(0,x) = \varphi_0(x), \quad u(0,x) = u_0(x), \; x \in \Omega \\ u(t,0) = u^0(t,x), \quad (t,x) \in R_- \times \Omega \end{cases}. \quad (2)$$

$$\begin{cases} \varphi(t,x) = \varphi_1(x) \\ u(t,x) + \alpha \dfrac{\partial u}{\partial \nu} 9t,x) = h(x), \quad x \in R_+ \times \Omega \end{cases}. \quad (3)$$

There, Ω is an open bounded subspace of R^N, ξ, τ, *l* – are positive constants, α ≥ 0 is a positive kernel, *f*, *g*: $R_+ \times \Omega \to R$, φ_0, u_0: $\Omega \to R$, u_0 : $R_- \times \Omega \to R$, and φ_1, *h*: $\Gamma \to R$ are given functions, and ∂/∂ν is the normal oriented to inward. In a classical Stefan's problem, this process is governed by equation:

$$(u + \frac{l}{2}\varphi(u))_t = k\Delta u, \quad in\ R \times \Omega. \quad (4)$$

where *u*(t) = *u*(*t*, *x*) is the temperature distribution of the region occupying Ω region and may be in one of the two phases, solid and liquid (if the melting temperature is made 0 degree), *K* is the thermal diffusion, that is the thermal conductivity of the heat capacity per unit volume (taking heat capacity per unit volume equal to unity), which for simplicity may be assumed to be the same in solid and liquid, and *l* is the latent heat.

In order to made differentiation in (4), let us suppose that internal energy *e* and heat flux *q* is given by the equation:

$$e = u + \frac{l}{2}\rho(u), \quad q = -k\nabla_x u, \quad (5)$$

and the heat balance is satisfied,

$$e = \nabla_x q. \quad (6)$$

If instead of Fourier law we use the constitutive equation with memory for heat flux, we obtain:

$$q(t,x) = -\int_{-\infty}^{t} a(t-s)\nabla_x u(s,x)dx, \quad (t,x)\in R_+ \times \Omega \,. \quad (7)$$

and using (5)−(6) we obtain:

$$(u+\frac{l}{2}\varphi(u))_t(t,x) = -\int_{-\infty}^{t} a(t-s)\Delta u(s,x)dx\,, \quad (8)$$

$$(t,x)\in R_+ \times \Omega \,.$$

In a particular case for a and $u = 0$ for $t < 0$, the equation (7) becomes:

$$a(t) = K_0 e^{-\varepsilon t}, \quad K_0, \varepsilon_0 > 0\,, \quad (9)$$

$$\frac{\partial^2}{\partial t^2}(u+\frac{l}{2}\varphi(u)) + \varepsilon \frac{\partial}{\partial t}(u+\frac{l}{2}\varphi(u)) = K_0 \Delta u\,. \quad (10)$$

A simplified variant is used for the classic study of problem:

$$\frac{\partial^2 u}{\partial t^2} + \varepsilon \frac{\partial}{\partial t}(u+\frac{l}{2}\varphi(u)) = K_0 \Delta u\,. \quad (11)$$

According to classic theory Landau-Ginzburg, φ depends on temperature u by equation (1). By the other hand, (7) and phasic field equation $\varphi(t, x)$ instead of $\varphi(u)$ conduct to second equation in (1). The conditions at limit $\alpha = 0$, $\varphi_1 = 1$, $u = h$ show that Γ is in a liquid phase at temperature h, meanwhile $\alpha = 0$ correspond to heat transfer process throughout Γ. E.g., taking into account (7) and u for $t < 0$, the heat transfer law is given by:

$$\int_{-\infty}^{t} a(t-s)(u+\alpha\frac{\partial u}{\partial v})(s,x)ds = r(t,x). \quad (12)$$

In can be proved the equation (1) has a unique solution and the solution is asymptotic for $t \to \infty$.

In the phase field model, the fracture is indicated by a scalar order parameter, which is coupled to the material properties, in order to shape change material stiffness between broken and unbroken zones. At interface level between rupture and undamaged material, the order parameter interpolates smoothly between values assigned to different phases of material. The width of this transition zone, which surrounds the fracture stages, is controlled by a regularization parameter (Fig. 2):

$$\varphi(x) = \exp\left(\frac{-|x-a|}{l_0}\right). \quad (13)$$

Phase field method proved to be a very powerful tool to solve moving boundary methods when numerical method are used to solve the equation that describes the behavior of interface between materials Numerical methods with finite differences are used to solve the equation (1). In this paper, a 2D approach is considered and a finite difference method for discrete time domain is developed as in [14 and 15]. The main results are presented in the next section.

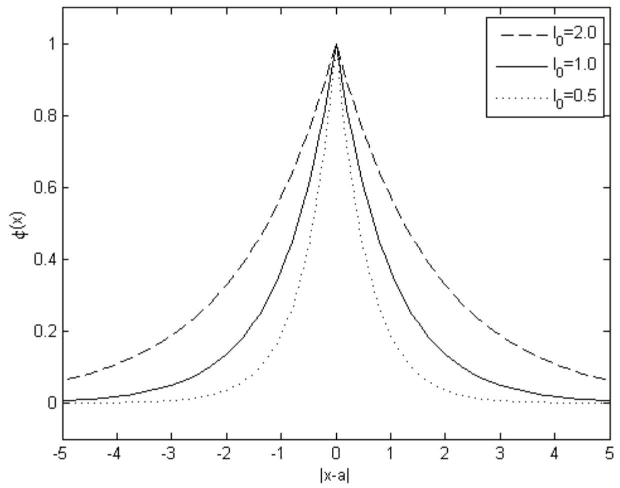

Fig. 2. Phase field of fracture φ for different levels of l_0.

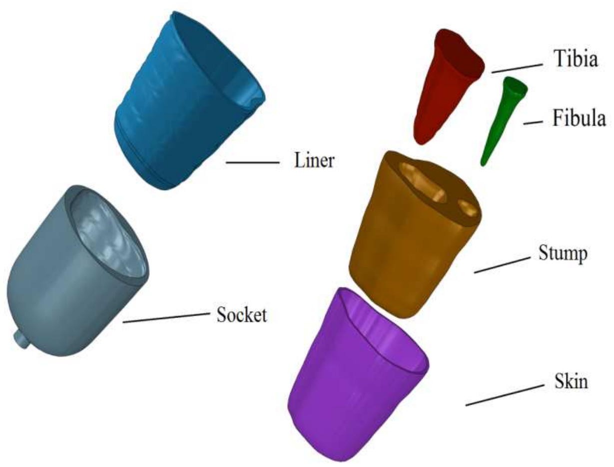

Fig. 3. The parts of modeled assembly stump-liner-socket.

3. EXPERIMENTAL RESULTS

Data were collected from a lot of two transtibial amputees, each one having a set of 32 repetition of cycle (8 cycles for normal walking, 8 cycles for up the stairs, 8 cycles for down the stairs and 8 cycles normal walking with no rotations). The walking cycle including rotation, walking on difficult terrain and up/down the stairs are the subject of the future research.

The stump is MRI scanned and NURB curves are used to approximate the 3D shapes. Detection contours algorithms are used to separate the various shapes involved in prosthesis: bones, skin, soft tissue, liner and cup.

Manual corrections were added when parts are inserted one in another in order to avoid incompatible mesh in FEA analysis. The bodies are inserted one into another taking into account the friction coefficients between surface. The bones and soft tissue are modeled as bonded bodies and skin and soft tissue are modeled as tied bodies (friction coefficient 1.0). The coefficient of friction between skin and liner is set to a value between 0.5−0.7 depending on the material of liner. The properties of skin are modeled to be the same for all the patients without customization. The coefficient of friction between cup (socket) and liner is set to a value

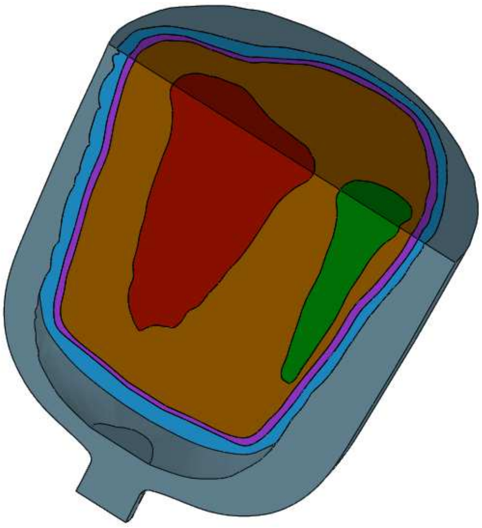

Fig. 4. The assembly cup-stump-liner in section.

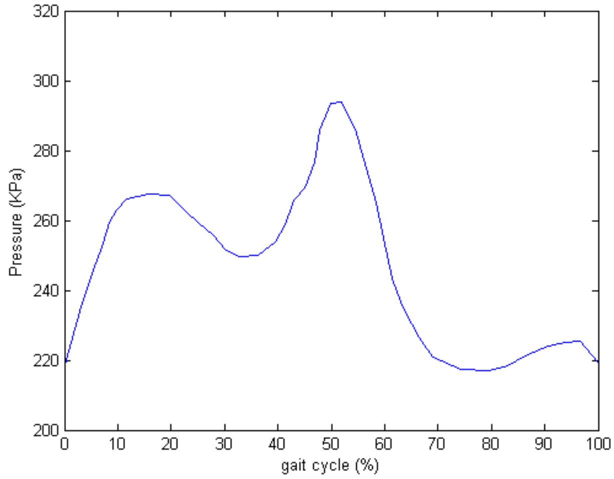

Fig. 5. Pressure in popliteal depression (PD).

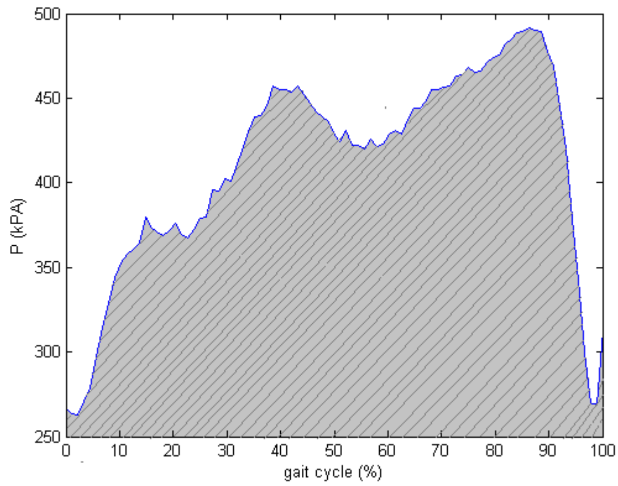

Fig. 6. Pressure record by sensor in location 6 (experimental results).

between 0.7−0.8 depending both of liner and cup materials.

The soft tissues is supposed to be a material that have the hyperelastic, homogeneous, and isotropic properties and they are modeled using the Generalized Mooney-Rivlin Solid strain energy function [15]:

$$W = C_{10}(I_1 - 3) + C_{11}(I_1 - I_3) + (1/D_1)\cdot(J - 1)^2. \quad (14)$$

The liner is soft Pelite material (Young's module = 0.38 MPa, Poisson coefficient = 0.39) and the cup is PP/PE (90% Polypropylene, 10% ethylene), Young's module = 1.5 GPa, Poisson coefficient = 0.3).

The pressure sensors are located (approximately) in the areas of most stressed areas based on FEM simulation in each case. The most stressed area is possible to be other that the four basic positions: (a) Patella tendon; (b) Popliteal depression; (c) Lateral tibia; and (d) Medial tibia (Fig. 4).

Even these locations are known to be solicitous areas, from pressure map during gait cycle, some areas can be susceptible to liner fractures. During wear, the stump donning into liner are slipping with very small displacement and create local friction with cup. These frictions along with friction between liner and stump increase, event very little in a single gait cycle the local temperature for the same area and cascading, the increasing friction coefficient (there are known nonlinear relationships between friction coefficient and temperature) that increase the temperature again. Even these evolutions are very small, if these actions take place a long period of time, they can produce damage in liner, practically a crack.

Our study propose a method to identify the most mechanical solicited areas and even to create a hierarchy of these locations under cycling loading. The loading cycle vary from cycle to cycle for the same person in different moment of walking, or different days. We have been chosen an average value with for n cycles (n = 120) recorded in similar conditions for the same person. The peaks (Fig. 5) happen for a short period of time, so we propose a more adequate measure, the sum of mechanical work in the selected stress areas (Fig. 6).

In return, we can see that the waveforms can have very different forms, depending on record location. In our approach we used six sensors distributed along the stump uniformly. In a second experiment, we placed the sensor in locations where the peaks of pressure are the highest. The third experiment considers the mechanical work calculated by formulas (15)−(17).

The area of sensor that records the pressure is known and the entire small area that surrounds the sensor is supposed to have the same value of pressure at one given moment of time. The formulas used are the classic ones, as in:

$$L_F = \int_{x_0}^{x_f} \vec{F} \cdot d\vec{x} = \int_{x_0}^{x_f} F \cos\alpha \cdot dx\,, \quad (15)$$

$$F = P \cdot A,\ x = v \cdot t\,, \quad (16)$$

$$L_F = A \cdot \int_{x_0}^{x_f} P dx\,. \quad (17)$$

The Caginalp phase field system (based on the Ginzburg-Landau theory of phase transition) has not been used yet for fracture modeling [12−14]. In this stage of research, we approximate the small rectangle selected from liner that have variable surface (Fig. 7) with a selected rectangle from a cylindrical shell.

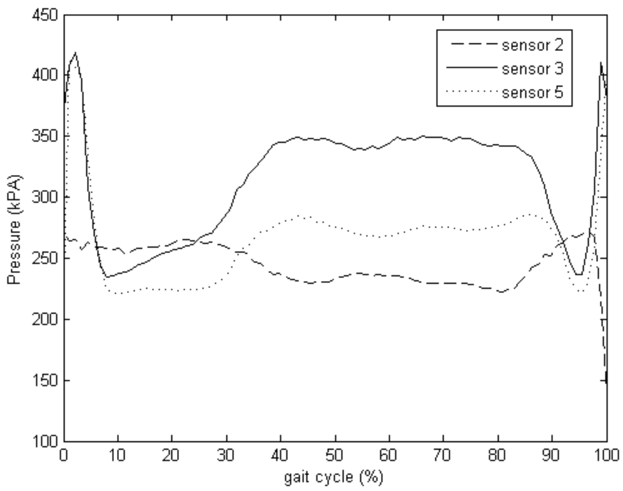

Fig. 7. Pressure record by sensor in location 2, 3, and 5 (experimental results).

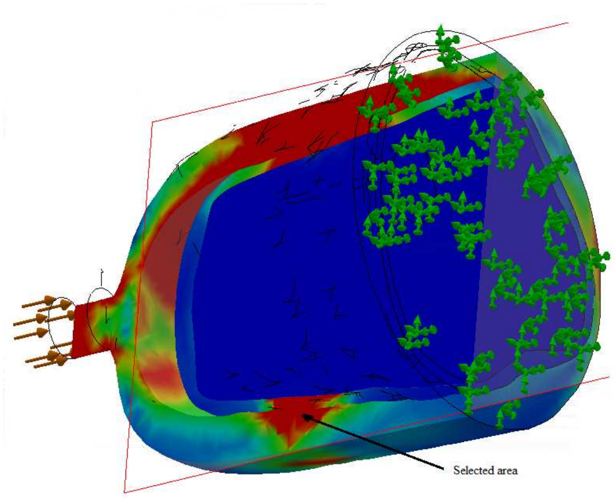

Fig. 8. Map of pressure on stump liner interface in section with selected area for crack development.

Fig. 9. 2D shell used in modeling of crack propagation using cycling loading.

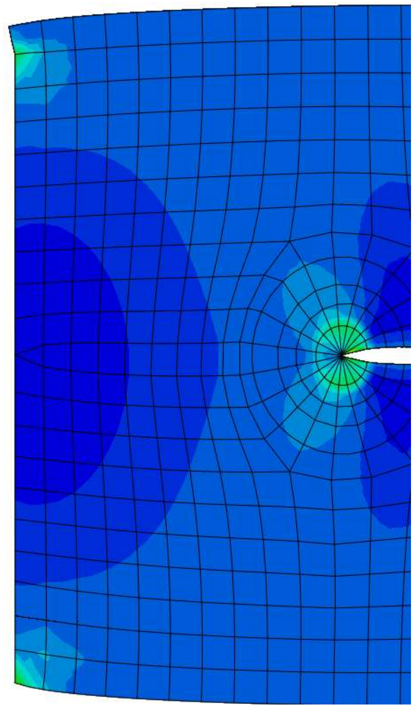

Fig. 10. Crack propagation on 2D surface (left half-plane).

There are some numerical solutions that are proposed by some authors, most of them being based on finite element methods, moving grids, discrete Fourier transform or finite difference method.

We have been chosen a combination of finite difference methods: forward difference, central difference and Crank–Nicolson method.

Two patients with unilateral transtibial amputation are subject to experiments. The subject A has 34 years old, 74 kg and subject B has 29 years, 88 kg.

The results are presented only for a liner used by patient A, in a cycling loading, during 1 200 000 times. In order to accelerate the visualization of crack development a proportional overloading was used that is not very realistic situation but the time required for simulations must be a feasible one. In the future research we plan to use a model of crack that takes into account the temperature development due friction also.

4. CONCLUSIONS

Finite element method was used to conduct a stress analysis and show the force distribution along the parts in a stump-liner-cup assembly. A method to quantify the most susceptible areas to damage due to pressure and friction was proposed in order to evaluate the crack prediction in liner. Convergence of phase field model in the corners and more generally to its sharp limits is still a problem for proposed method but in the future research we will apply an method adapted from known one developed for Cahn-Hilliard one.

The results are presented for a particular 2D case. In the future research a 3D solution will be developed.

Some details have not taken into account in this study. A composite material with different Young's modulus can be a solution for a liner that is very resistant to wear. Other aspects, such as stump deformation in time will be also a subject of research. The results contribute to a better understanding the design of an optimized prosthesis that increases the patient's performance along with a god choice of liner, made by an appropriate material that fits better to a particular blunt. The study of prosthetic application is an exciting and important topic in research and will profit considerably from theoretical input.

The result interpretation has been a permanent collaboration between math's and medical orthopedics.

REFERENCES

[1] S. Ali, *Clinical investigation of the interface pressure in the trans-tibia socket with Dermo and Seal-in X5 liner during walking and their effect on patient satisfaction*, Clinical Biomechanics , Vol. 27, 2012, pp. 943−948.

[2] M. C. Faustini, R. R. Neptune, R. H. Crawford, *The quasi-static response of compliant prosthetic sockets for transtibial amputees using finite element methods*, Medical Engineering & Physics, Vol. 28, 2006, pp. 114−121.

[3] S. Portnoy, *Internal mechanical conditions in the soft tissues of a residual limb of a trans-tibial amputee, Journal of Biomechanics*, Vol. 41, 2008, pp. 1897−1909.

[4] G. Simpson, et. all., *Modelling the interaction between a prosthetic socket, polyurethane liners and the residual limb in transtibial amputees using non-linear finite element analysis*, Biomedical Science Instrumentation, Vol. 37, 2001, pp. 343−347.

[5] A. Buis, P. Convery, *Calibration problems encountered while monitoring stump/socket interface pressures with interface force sensing resistors: techniques adopted to minimize inaccuracies*, Prosthetics and Orthotics International, Vol. 21, 1997, pp. 179−182.

[6] J. van Kuilenburg, M.A Masen, E. van der Heide, *Contact modelling of human skin: What value to use for the modulus of elasticity?*, Proc IMechE Part J: J Engineering Tribology, 2012, pp. 1−13.

[7] C. Miehe, L. Schanzel, H. Ulmer, *Phase field modeling of fracture in multi-physics problems. Part I. Balance of crack surface and failure criteria for brittle crack propagation in thermo-elastic solids*, Comput. Methods Appl. Mech. Engrg., 2014.

[8] M. A. Msekh, et. all., *Abaqus implementation of phase-field model for brittle fracture*, Computational Materials Science, Vol. 96, 2015, pp. 472−484.

[9] Y. Xu, J.M. McDonough, K.A. Tagavi, *A Numerical procedure for solving 2D phase-field model problems*, Journal of Computational Physics, Vol. 218, 2006, pp. 770−793.

[10] J. Cho, K.-S. Lee, *Finite element simulation of crack propagation based on phase field theory*, Journal of Mechanical Science and Technology, Vol. 27, No. 10, 2013, pp. 3073−3085.

[11] C. V. Verhoosel , R. de Borst, *A phase-field model for cohesive fracture*, International Journal for numerical methods in Engineering, 2010, pp. 1−20.

[12] G. Caginalp, X. Chen, C. Eck, *Numerical Tests of a Phase Field Model with Second Order Accuracy*, SIAM J. Appl. Math., Vol. 68, Issue 6, 2008, pp. 1518−1534

[13] G. Caginalp, J.T. Lin, *A numerical analysis of an anisotropic phase field model*, IMA Journal of Applied Mathematics, Vol. 39, Issue 1, 1987, pp. 51-66.

[14] G. Caginalp, P. Fife, *Phase-field methods for interfacial boundaries*, Phys. Rev. B, Vol. 33, Issues 11, 1986, pp. 7792−7794.

[15] M. Zhang, A. F.Mak, *A Finite Element Analysis of the Load Transfer Between an Above-Knee Residual Limb and its Prosthetic Socket-Roles of Interface Friction and Distal-End Boundary Conditions*, IEEE Transactions on Rehabilitation Engineering, Vol. 4, 1996, pp. 337−346

[16] N. W. Oren, *A Finite Difference Implementation of Phase Field Theory*, Master's Theses, University of Connecticut Graduate School, 2011.

[17] A. N. Mathioudakis, *Finite Element Formulation of Phase Field Fracture*, Master's Theses, University of Connecticut Graduate School, 2014.