Proceedings in MANUFACTURING SYSTEMS

Proceedings in Manufacturing Systems, Volume 13, Issue 1, 2018, 21-26 ISSN 2067-9238

THERMAL MODEL OF EXTERNALLY DRIVEN SPINDLE: A SEMI-AUTOMATIC CONSTRUCTION OF THERMAL NETWORK

Andrei GHEORGHIȚĂ[1], Dragoș AROTĂRIȚEI[2,*], George CONSTANTIN[3], Marius TURNEA[4], Mihai ILEA[5], Mariana ROTARIU[6]

[1)] PhD Student, Dept. Medical Bioengineering, University of Medicine and Pharmacy "Grigore T. Popa", Iasi, Romania
[2,*)] PhD. Prof., Dept. Medical Bioengineering, Univ. of Medicine and Pharmacy "Grigore T. Popa", Iasi, Romania
[3)] PhD, Prof., Machine and Production Systems Department, University "Politehnica" of Bucharest, Bucharest, Romania
[4)] Assistant Professor, Dept. Medical Bioengineering, Univ. of Medicine and Pharmacy "Grigore T. Popa", Iasi, Romania
[5)] Assistant Professor, Dept. Medical Bioengineering, Univ. of Medicine and Pharmacy "Grigore T. Popa", Iasi, Romania
[6)] Assistant Professor, Dept. Medical Bioengineering, Univ. of Medicine and Pharmacy "Grigore T. Popa", Iasi, Romania

Abstract: *This paper. Focuses on one application to construct a user guided thermal network for externally driven spindle of high speed. The objective is to develop a model the can be used for effective estimation of thermally distribution in a spindle-bearing system using and generator of linear symbolic system of equations that can be solved using a numerically solver in a simpler manner than the tools that use the finite element model. The heat generated by bearings is calculated using a dynamic model of angular contact bearings and cylindrical roller bearings in a distribution and construction that use preload. The results of simulation were compared with similar results from literature and for a moderate level of granulation of thermal network the results were found good for practical applications. The proposed approach is useful especially when we need in a fast manner practical results and the number of node must be find out by multiple iterations with less ore more complex thermal network architecture. The prospective development and further research are discussed.*

Key words: *Spindle-bearing modeling, thermal network, heat transfer, thermal resistance, angular contact ball bearing, system of linear equations.*

1. INTRODUCTION

The accuracy and lifetime of a rotational machine tool is conditioned by its static, dynamic, thermic, vibrational and thermo-elastic behavior [1]. Main spindle-bearing system contributes to final accuracy of workpiece and the quality of it determines in general the performance of the machine. The thermal behavior of machine and the predictability of it thermic evolution in dynamic regime is an important issue for a good function of rotated machines. The stiffness and deformations of spindle contribute to accuracy of the manufactured piece.

Simulations can reduce substantially the design effort for a performing machine versus an approach based on experimental studies [2]. The simulations can offer a better understanding of interaction among components among with possible optimization of location of bearings and preload values for contact angle bearings. Other design aspect can influence the thermal behavior of the systems as the house, the cooling subsystem and the motor placement. Two main approaches are related to motor placement: built-in motor and externally driven spindle, usually by a belt. These two approaches influence the types of heat sources that are taken into account. In the built-in case, the main source of heat is the electric engine itself and the bearings meanwhile in the second case, the main source of the heat are practice the bearings. The other secondary heat sources are negligible, in the most of the research papers.

The friction that occurs in the bearings is the main source of the heat [1]. As the speed increase, the heat generated in the bearing and the cutting surface will increase. The Finite Element Method (FEM) is widely used to model dynamic, thermic and thermo-mechanic behavior of the spindle-bearing system [3–5]. The method offer accurate prediction of thermal behavior but it is expansive in calculation time and complexity [6]. The main drawback of the method is that it need a lot of time to construct the model and it cannot be applied to different dynamic regimes of work.

Thermal network [7] has proven to be a good approximation for thermic analysis of heat transfer phenomena [8 and 9]. In [10], the authors proposed a mathematical model for ball friction and used it for heat generation, heat transfer, and transient temperature distribution and different speeds. The model was validated by experimental results.

A unified method based on FEM and thermal resistance for a quasi-static model was proposed in [11] in order to predict the thermal characteristics of high speed spindle subject to preload.

The parameters of angular contact bearings were considered: thermal contact resistance, geometric dimensions, lubricant viscosity and thermal effects on the contact angle between ball and rings and ball bearing [11]. The nonlinear behavior of transient temperature is verified experimentally.

[*] Corresponding author: Kogalniceanu str., no. 9-13, Iasi, Romania,
Tel.: +40.232.213.573;
Fax: +40.232.213.573
E-mail addresses: *dragos.arotaritei@gmail.com* (D. Arotăriței).

Numerical solutions for equations the describe Steady-state temperature distribution including load-deflection analysis and calculation of thermal expansion were proposed in [12]. Formulas for thermal resistance of spindle, bearings and housing are also described in detail [12].

The Hertz's contact theory was applied to model of spindle-bearing system along elastohydrodynamic lubrication (EHL) theory for a thermo-mechanical analysis was proposed in [13]. The spindle cooling system effect was taking into account for steady-state heat balance between heat generation and cooling elements [13].

A more sophisticated approach was presented in [14]. A thermal network that includes oil-air lubrication and convection effect inside the cage was used to predict thermal behaviors of high-speed angular contact ball bearings [14]. Detailed formulas for heat convection were presented and the influence of them is included in a multi-node model thermal network for angular contact bearing [14].

2. DYNAMIC MODEL OF BEARING AND HEAT GENERATION

The accuracy and lifetime of a rotational machine tool is conditioned by its static, dynamic, and thermic behavior. The friction between balls and raceway depend on dynamic load on each ball depend and the stiffness that also depends on the contact angle [1].

As sequel, the heat generated bearings must start with dynamic analysis of rolling bearings and the total heat, considered in a center of bearing is the sum of heat produced by each ball.

In our approach, the ball bearing is the subject of centrifugal force F_{cj} and gyroscopic momentum M_{gj}, where j is the jth position $j = 0, 1, ..., Z - 1$ for each one of Z balls at $\psi = (j \cdot \pi) / (Z - 1)$ angle. The misalignment angle θ is very small in comparison with other variable and it was neglected in calculus. Let us denote by d_i – inner raceway diameter, d_0 – inner raceway diameter, d_m – bearing pitch diameter, D – ball diameter, r_i − inner groove radius and; r_o − outer groove radius.

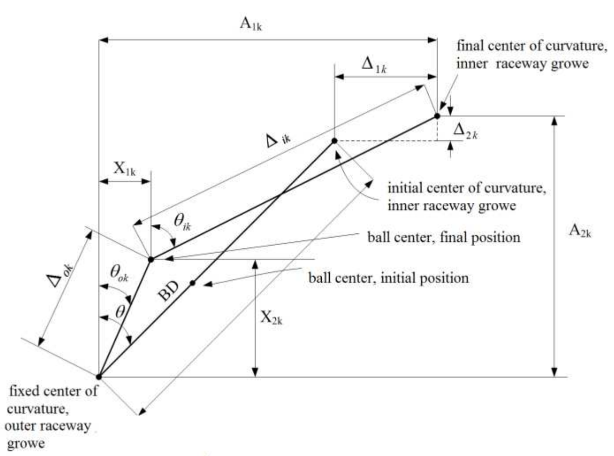

Fig. 1. Position of ball center after preload application and displacements for angular contact ball bearing.

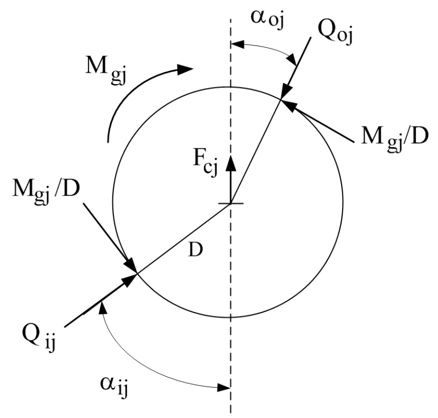

Fig. 2. Ball loading and main forces that act on it at ψ_j angle.

The Hertzian contact forces between balls and raceways are given by formulas [15]:

$$Q_i = K_i \delta^n, \quad Q_o = K_o \delta^n . \tag{1}$$

where n = 3/2 for angular contact bearings and 10/9 for cylindrical rolling bearing and i stands for inner meanwhile o stands for outer. Due to lack of space we will present in shorter manner the calculation of Q_{ij} and Q_{oj} based on [15 and 16] approaches. K is a stiffness axial constant and it is calculated from tables [15]. However, the table is not always available and approximated formulas using regressive models are often used [17 and 18]. Let us denote by A and B the two elastic bodies in contact:

$$K = \frac{\pi k E'}{3F}\sqrt{\frac{2\varepsilon R}{F}}; \tag{2}$$

$$k = 1.0339\left(\frac{R_y}{R_x}\right)^{0.6360}; \tag{3}$$

$$F = 1.5277 + 0.6023 \ln\left(\frac{R_y}{R_x}\right); \tag{4}$$

$$\varepsilon = 1.0003 + 0.5968\left(\frac{R_x}{R_y}\right); \tag{5}$$

$$E' = 2 / \left((1-\nu_A^2)/E_A + (1-\nu_B^2)/E_B\right). \tag{6}$$

In formulas (2)–(6), E is the Young's modulus; ν – the Poisson's constant; R – sum of curvatures, $1 / R = 1 / R_x + 1 / R_y$; R_x – curvature radius in x plane; R_y – curvature radius in y plane in the contact point [15]. For inner ring and outer ring respectively:

$$R_x = \frac{D}{2}(1-\gamma_i), \quad R_y = D\frac{f_i}{2f_i - 1}; \tag{7}$$

$$\gamma_i = D\cos\alpha_{ij} / d_m . \tag{8}$$

$$R_x = D\frac{f_o}{2f_o - 1}, \quad R_y = \frac{D}{2}(1+\gamma_o); \tag{9}$$

$$\gamma_o = D\cos\alpha_{oj} / d_m . \tag{10}$$

The equations that describe the mathematical relations from Fig. 1 are given in following equations and the approach is similar to [15]:

$$A_{1j} = BD\sin\alpha^0 + \delta_a\,; \tag{11}$$

$$A_{2j} = BD\sin\alpha^0 + \delta_r\cos\psi_j\,; \tag{12}$$

$$\cos\alpha_{oj} = \frac{X_{2j}}{(f_0 - 0.5)D + \delta_{oj}}\,; \tag{13}$$

$$\sin\alpha_{oj} = \frac{X_{1j}}{(f_0 - 0.5)D + \delta_{oj}}\,; \tag{14}$$

$$\cos\alpha_{ij} = \frac{A_{2j} - X_{2j}}{(f_0 - 0.5)D + \delta_{ij}}\,; \tag{15}$$

$$\sin\alpha_{ij} = \frac{A_{1j} - X_{1j}}{(f_0 - 0.5)D + \delta_{ij}}\,. \tag{16}$$

Using trigonometric relation $\sin^2 x + \cos^2 x = 1$ and the relation $f = r / D$, one can write:

$$(A_{1j} - X_{1j})^2 + (A_{2j} - X_{2j})^2 - \Delta_{ij}^2 = 0\,; \tag{17}$$

$$X_{1j}^2 + X_{2j}^2 - \Delta_{oj}^2 = 0\,; \tag{18}$$

$$\Delta_{ij} = (f_i - 0.5)D + \delta_{ij}\,; \tag{19}$$

$$\Delta_{oj} = (f_o - 0.5)D + \delta_{oj}\,. \tag{20}$$

The conditions of equilibrium from each jth ball from all Z balls are given by:

$$\begin{aligned} &Q_{ij}\cos\alpha_{oj} - \frac{M_{gj}}{D}\sin\alpha_{oj} - Q_{ij}\cos\alpha_{ij} + \\ &\frac{M_{gj}}{D}\sin\alpha_{ij} - F_{cj} = 0 \end{aligned}\;; \tag{21}$$

$$\begin{aligned} &Q_{oj}\sin\alpha_{oj} + \frac{M_{gj}}{D}\cos\alpha_{oj} - Q_{ij}\sin\alpha_{ij} \\ &- \frac{M_{gj}}{D}\cos\alpha_{ij} = 0 \end{aligned}\;. \tag{22}$$

In the developed model it is considered that the inner ring is rotating with the speed n [rpm], meanwhile the outer ring is mounted on the housing. The angular speed is given by $\omega = 2\pi n / 60$. The centrifugal force and gyroscopic momentum are equal with:

$$F_{cj} = \frac{1}{2} mD\omega^2 \left(\frac{\omega_E}{\omega}\right)_j^2; \tag{23}$$

$$M_{gj} = J_b\omega^2 \left(\frac{\omega_B}{\omega}\right)_j \left(\frac{\omega_E}{\omega}\right)_j \sin\alpha_j\,; \tag{24}$$

$$\frac{\omega_E}{\omega} = \frac{\cos(\alpha_{ij} - \alpha_{oj}) - D/d_m\cos(\alpha_{oj})}{1 + \cos(\alpha_{ij} - \alpha_{oj})}\,; \tag{25}$$

$$\tan\alpha_j = \frac{\sin\alpha_{ij}}{\cos\alpha_{ij} - D/dm}\,; \tag{26}$$

$$\begin{aligned} \frac{\omega_B}{\omega} = -1/\Bigg(&\frac{\cos\alpha_{oj} + \tan\alpha_j\sin\alpha_{oj}}{1 + D/d_m\cdot\cos\alpha_{oj}} + \\ &\frac{\cos\alpha_{ij} + \tan\alpha_j\sin\alpha_{ij}}{1 - D/d_m\cdot\cos\alpha_{ij}}\Bigg)(D/d_m)\cos\alpha_j \end{aligned}\,. \tag{27}$$

The four equations (17), (18), (21) and (22) have as unknown values $\{P\} = \{X_{1j}, X_{2j}, \delta_{0j}, \delta_{ij}\}^T$.

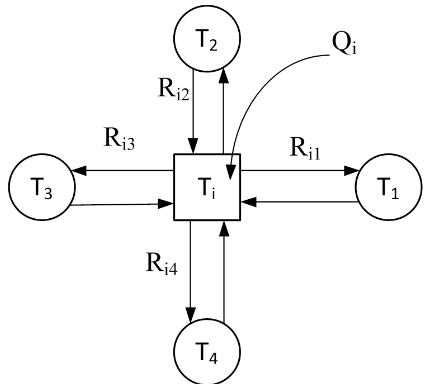

Fig. 3. Thermal point associate with a node and its adjoining nodes.

The most common solution is the Newton-Raphson method that uses successive iterations and the Jacobian of system of nonlinear equations (or finite difference if the inter-dependencies between variables and additional variables are too complicated. The errors of system of equations are:

$$(A_{1j} - X_{1j})^2 + (A_{2j} - X_{2j})^2 - \Delta_{ij}^2 = q_1\,; \tag{28}$$

$$X_{1j}^2 + X_{2j}^2 - \Delta_{oj}^2 = q_2\,; \tag{29}$$

$$\begin{aligned} &Q_{ij}\cos\alpha_{oj} - \frac{M_{gj}}{D}\sin\alpha_{oj} - Q_{ij}\cos\alpha_{ij} + \\ &\frac{M_{gj}}{D}\sin\alpha_{ij} - F_{cj} = q_3 \end{aligned}\;; \tag{30}$$

$$\begin{aligned} &Q_{oj}\sin\alpha_{oj} + \frac{M_{gj}}{D}\cos\alpha_{oj} - Q_{ij}\sin\alpha_{ij} \\ &- \frac{M_{gj}}{D}\cos\alpha_{ij} = q_4 \end{aligned}\;. \tag{31}$$

Equations are solved iteratively, and at $n + 1$ step, the values are given from precedent step:

$$\{P_j^{\,n+1}\} = \{P_j^{\,n}\} - [a_{ij}]^{-1}\{q_j^n\}, \quad n = 0, 1, ..., n;\; j = 1, 2, 3, 4. \tag{32}$$

The iterations continue until the predetermined convergence criteria are meet, or the no matter how many iterations are made, the maximum error does not decrease. The coefficients a_{ij} are those from [16].

3. THE THERMAL NETWORK

Heat transfer model is used to compute the distribution of temperature in a spindle bearing system. Thermal networks associated parts of the machine with a linear thermal resistance, each part connected with its neighbors governed by equation of thermic equilibrium in a manner similar with Kirchhoff's laws from electrical circuits [7].

The structure is discretized and each block from this structure is associated with a node (Fig. 3), the center of each element is described as a location having the average temperature for entire element. The heat Q from this center is transferred to adjacent center of nodes via contact areas.

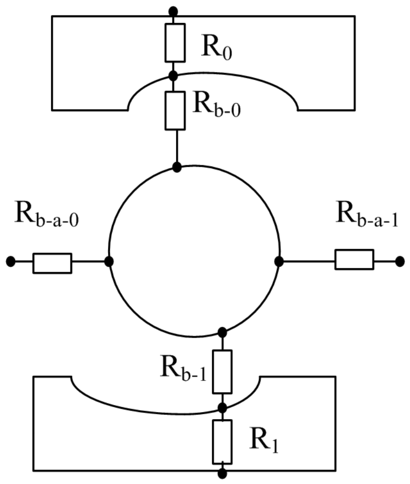

Fig. 4. Seven points model of bearing: R_0, R_1 – thermal resistance bearings, a – index for air, b – index for bearing.

The equation associated with node from Fig. 3 is given by [8]:

$$\sum_{k=1}^{4} \frac{T_k - T_i}{R_{ik}} + Q_i = m_i c_i \frac{\partial T_i}{\partial t}, \quad (33)$$

where T_k is the temperature of k node, $k = \{1, 2, 3, 4\}$, T_i – temperature of i node; R_{ik} – thermal resistance between node i and k; Q_i, c_i and m_i – heat rate, thermal capacity of node i and mass, respectively.

The system of equations is made by energetic balance: applied to each node of the system: the input energy + output energy = 0, and the usually ambient temperature is considered T_a = 25 C°. Finally, it will be obtained a linear system of equations that looks like that described by (34) for a purely hypothetic network with two sources of heat, Q_1 and Q_2 and five nodes, the last one being connected to ambient. The equation are useful for demonstrating the symbolic calculus constructed by equation generator in our proposed software tool:

$$\begin{cases} Q_1 - \frac{T_1 - T_2}{R_{12}} - \frac{T_1 - T_3}{R_{13}} - \frac{T_1 - T_4}{R_{14}} = 0 \\ Q_2 + \frac{T_1 - T_2}{R_{12}} - \frac{T_2 - T_a}{R_{25}} = 0 \\ \frac{T_1 - T_3}{R_{13}} - \frac{T_3 - T_4}{R_{34}} - \frac{T_3 - T_a}{R_{35}} = 0 \\ \frac{T_1 - T_4}{R_{14}} + \frac{T_3 - T_4}{R_{34}} - \frac{T_4 - T_a}{R_{45}} = 0 \end{cases}. \quad (34)$$

In matrix form, the system of equations becomes $[R]\{T\} = Q$ with solution $\{T\} = [R]^{-1} Q$, solution that can be calculated directly (or by substitution, or by Cramer) if the matrix R is non-singular or by iterative approach, e.g. Gauss-Seidel method. If there are n nodes, a number of n equations need to be developed in order to calculate the final temperatures of each node.

The thermal phenomena in bearing are of three types [15]: conduction, convection and radiation. The radiation is very small, and in almost all refered articles this component is neglected. The model of the bearing is selected to be of 7-nodes type as in Fig. 4 [19]. The total heat generated by bearings as function of total torque M_t and speed n is given by [15]:

$$Q = 1.047 \times 10^{-4} n M_t, \quad (35)$$

$$M_t = M_l + M_v + M_s, \quad (36)$$

where M_l = moment due to load on ball, M_v – moment due to viscous friction and M_s – the spinning moment.

$$M_l = f_1 F_1 d_m, \quad (37)$$

$$M_v = \begin{cases} 10^{-7} \cdot (\upsilon_o n)^{2/3} f_v d_m^3 & if\ \upsilon_o n \geq 2000 \\ 10^{-7} \cdot 160 \cdot f_v d_m^3 & if\ \upsilon_o n < 2000 \end{cases}, \quad (38)$$

$$M_s = \frac{3 \mu Q a E}{8}, \quad (39)$$

where f_1 represents the a factor related to bearing type, F_1 – static equivalent load, f_v – factor that depend on the bearing and lubrication type, ν_0 – kinematic viscosity (depending of temperature and lubricant), μ – friction coefficient (the most common value is 0.6), Q – normal contact force, a – semi-major axis in ellipse that describe the contact between ball and grove; E – elliptical integral of second order used to calculate the stiffness of bearing [15].

Linear thermal resistance can be calculated by definition [20]:

$$R = L / KA, \quad (40)$$

where L is the length of element, A the cross-sectional area and K thermal conductivity that depend on material. The spindle-bearing system is considered by approximation to be a symmetrical one, and the basic solid elements that can be parts of this system for thermic analysis are: hollow cylinder, cylinder and tapered cone (hollow). The detailed formulas and notations for thermal source location of bearing are presented in [20]:

$$R_{radial} = \frac{\ln(r_o / r_i)}{2\pi k L}, \quad R_{axial} = \frac{\Delta x}{kA}; \quad (41)$$

$$R_b = \frac{1}{k_b \pi r_b}; \quad (42)$$

$$R_{Li} = \frac{r_b}{k_l \left(\frac{2\pi}{n} r_i W_i - \pi r_b^2 \right)}; \quad (43)$$

$$R_{Lo} = \frac{r_b}{k_l \left(\frac{2\pi}{n} r_e W_e - \pi r_b^2 \right)}. \quad (44)$$

The contact between inner ring and shaft is practically a joint made under pressure, creating so name the contact resistance coefficient from which it can be deduced the thermal contact resistance, $r_c = 1 / h$, [21 and 22]. Detailed calculus for situations, plastic deformation and elastic deformation can be found in [22].

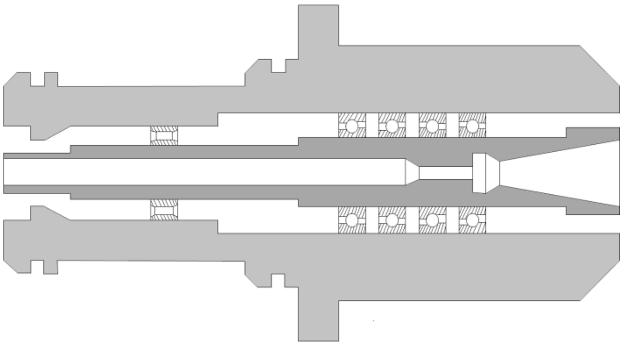

Fig. 5. The formal sketch of spindle bearing system.

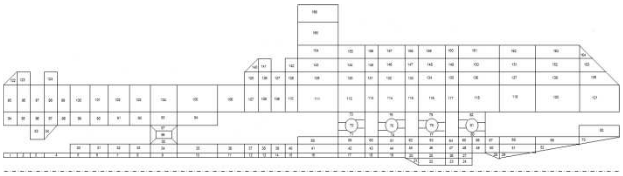

Fig. 6. Meshed spindle-bearing in *m* elements (*m* = 165).

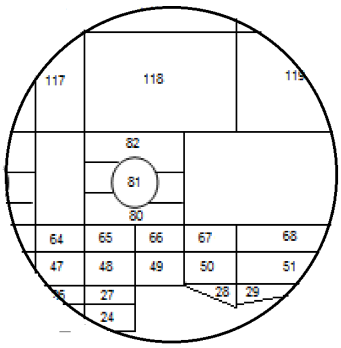

Fig. 7. Circular zoom of bearing area from Fig. 6.

3. SOFTWARE TOOL FOR SEMI-AUTOMATIC CONSTRUCTION OF THERMAL NETWORK ASSOCIATED WITH A SPINDLE-BEARING

The experiments were made using a spindle-bearing high speed grinding machine available in mechatronic laboratory. The spindle has four contact angle bearings in tandem and one cylindrical rolling bearing. The formal sketch is given in Fig. 5.

The core model of our software tool is the equation generator. The section in the spindle-bearing (Fig. 5), is split manually in *m* regular elements (rectangle, circle and trapezoidal), in a symmetrical half-section. The operation is made on an image file (in our case the .png, .bmp and .jpg are the acceptable formats) .A crosshair help the user to allocate a number to each element by a double click on approximate center of element. A control module keep track on the counter of element numbers, the delete, modify or append a new number. A geometrical sub-module asks for air-bearing or T_{amb} interconnection for selected modules. The generator of schema module save this graphic processed image in en Excel type file that have all the information about one element: number, material, the neighbors, generator of heat, form of element and dimensions.

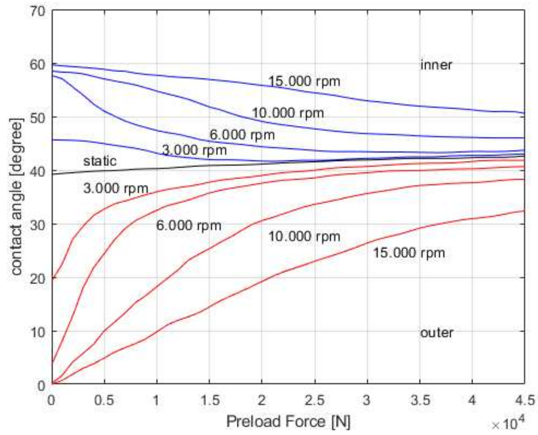

Fig. 8. Angle of contact vs. preload force.

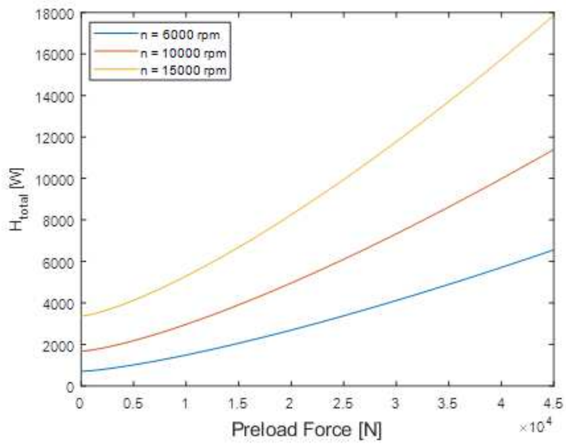

Fig. 9. Heat total vs. preload force, angular contact bearing.

A parser that work with symbolic toolbox from MATLAB take into account each node, and four one equation written by thermic equilibrium in the node is translate as $T = \{T_1\ T_2\ T_3\ T_4\}^T$ in example (34), *R* matrix and the *Q* vector. All R_i matrices are concatenated, $i = 1, 2, \ldots, m$ in a final *R* matrix in order to solve the equation $\{T\} = [R]^{-1}\{Q\}$.

The entire program is written in MATLAB 2017b, and the graphical interface is made by using Guide. The user must write in Excel file the dimensions of each element along with characteristic of material if the material is not predefined steel. Based on geometric characteristics, the thermal resistance for each solid body (element) necessary to solve the thermal network is calculated using (41)–(44) and [22]. The user can extract the temperature value for a single node or for a group of nodes, e.g. a line along the spindle.

4. EXPERIMENTAL RESULTS OF SIMULATIONS

Our model does not consider the thermal expansion of bearings as part of their deflections. The analysis of thermal properties occurs in this assumption independently of load-deflection study.

For a 219 bearing that equip the spindle [15], the angle of deflection and heat generation is presented in Figs. 8–9.

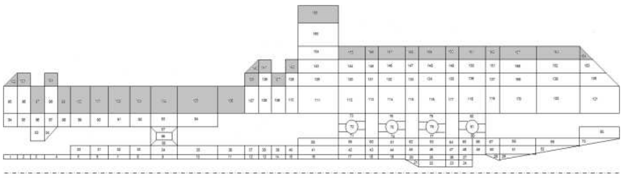

Fig. 10. Profile A selected to display temperatures.

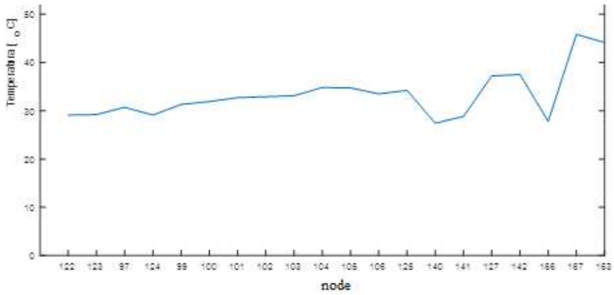

Fig. 11. Temperature curve, profile A, stable thermal status.

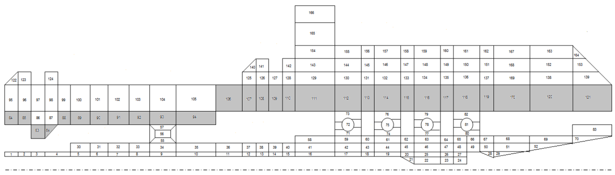

Fig. 12. Profile B selected to display temperatures.

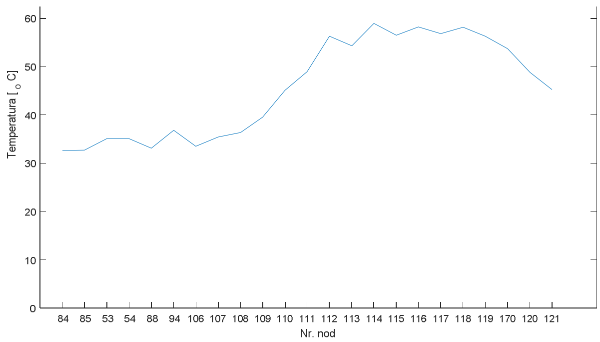

Fig. 13. Temperature curve, profile B, stable thermal status.

Simulation results with temperatures in °C along a line selected by user are presented in Figs. 10−13.

5. CONCLUSIONS

The software tool gives satisfactory results in this preliminary stage but some improvements should be done. An automatic geometric selection of dimensions must be done. Also the automatic split in element of the 2D sketch of spindle bearing will be made in the future development. An interface with for calculus in of thermal resistance is in progress.

The difficulties that are encountered in the graphical user interface in MATLAB tool require more time to develop an automatic tool in the CAD (Computer Aided Design) tool style of a multiphysics tool.

REFERENCES

[1] C. Brecher, A. Wissmann, *Compensation of Thermo-Dependent Machine Tool Deformations Due to Spindle Load: Investigation of the Optimal Transfer Function in Consideration of Rough Machining*, Production Engineering, Vol. 5, 2011, pp. 565−574.

[2] J. Mayr, et al., *Thermal Issues in Machine Tools*, CIRP Annals - Manufacturing Technology, Vol. 61, 2012, pp. 771−791.

[3] B. Bossmans, J. Tu, *Thermal Model for High Speed Motorized Spindles*, International Journal of Machine Tools and Manufacture, Vol. 39, 1999, pp. 1345−1366.

[4] H. Li, Y.C. Shin, *Integrated Dynamic Thermo-Mechanical Modeling of High Speed Spindles. Part 1: Model Development*, Journal of Manufacturing Science and Engineering, Vol. 126, 2004, pp. 148−158.

[5] J.D. Kim, I. Zverv, K.B. Lee, *Thermal Model of High-Speed Spindle Units*, KSME International Journal, Vol. 17, 2003, pp. 668−678.

[6] S.-M. Kim, S.-K. Lee, *Prediction of thermo-elastic behavior in a spindle–bearing system considering bearing surroundings*, International Journal of Machine Tools & Manufacture, Vol. 41, 2001, pp. 809–831.

[7] A. O. Gibson, J. L. Stein, J.F. Tu, *A Thermomechanical Model of High-Spindle Bearing Systems for use in the Design of Spindle Bearing Load Control Systems*, ASME Journal of Basic Engineering, 1960, pp. 309–320.

[8] T. L. Bergman, A.S. Lavine, F.P. Incropera, D.P. DeWitt, *Fundamentals of Heat and Mass Transfer*, 7th Edition, 2011, Wiley.

[9] Y.A. Cengel, A.J. Ghajar, *Heat and Mass Transfer: Fundamentals and Applications*, 5th Edition, 2014, McGraw-Hill Education.

[10] J. Takabi, M.M Khonsari, *Experimental testing and thermal analysis of ball bearings*, Tribol. Int., Vol. 60, 2013, pp. 93–103.

[11] V.-T. Than, J.H. Huang, *Nonlinear thermal effects on high-speed spindle bearings subjected to preload*, Tribology International, Vol. 96, 2016, pp. 361−372.

[12] B. R. Jorgensemn, Y. C. Shin, *Dynamics of Machine Tool Spindle/Bearing Systems Under Thermal Growth*, Journal of Tribology, Vol. 119, 1997, pp. 885−-882.

[13] L. Wu, Q. Tan, Thermal Characteristic Analysis and Experimental Study of a Spindle-Bearing System, Entropy, Vol. 18, 2016, pp. 271−296.

[14] D. Zhenga, W. Chena, Thermal performances on angular contact ball bearing of high-speed spindle considering structural constraints under oil-air lubrication, Tribology International, Vol. 109, 2017, pp. 593–601.

[15] T.A. Harris, Rolling Bearing Analysis, John Wiley & Sons, Inc, 4th Edition, 2001, pp 337−361.

[16] Y. Cao, Y. Altintas, *A General Method for the Modeling of Spindle-Bearing Systems*, Journal of Mechanical Design, Vol. 126, 2004, pp. 1089−1104.

[17] D. E., Brewe, B. J. Hamrock, *Simplified Solution for Elliptical- Contact Deformation Between Two Elastic Solids*, ASME J. Lubr. Technol., Vol. 99, 1977, pp. 485–487.

[18] J. A. Greenwood, *Analysis of Elliptical Hertzian Contacts, Tribol. Int.*, Vol. 30, 1997, pp. 235–237.

[19] D. Zheng, W. Chen, *Thermal performances on angular contact ball bearing of high-speed spindle considering structural constraints under oil-air lubrication*, Tribology International, Vol. 109, 2017, pp. 593–601.

[20] Bert R. Jorgensen, Yung C. Shin, *Dynamics of Machine Tool Spindle/Bearing Systems Under Thermal Growth*, Journal of Tribology, 1997, Vol. 119, pp. 875−882.

[21] *K Nakajima, Thermal contact resistance between balls and rings of a bearing under axial, radial, andc ombined loads, J. Thermophys*, Heat Transf., 1995, Vol. 9, pp. 88–95.

[22] C.V. Madhusudana, *Thermal contact conductance, Springer*, Switzerland, 2014.