CUTTING KINEMATICS ANALYSIS IN HYPOCYCLOIDAL TOOTH MILLING OF CYLINDRICAL GEARS

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Abstract: The hypocycloidal flanks of the cylindrical gear teeth have the flank lines defined by lengthened or shortened hypocycloid and the profile involute. The curved and bulged flanks have some advantages, among them being the increased strength in bending and controlled positioning of the contact imprint. The tooth flank generation has on the basis the application of the technological process of milling using a cutting tool with multi-cutters by rolling with mobile line and continuous division. In the paper, the components of the cutting motion and their speed are analytically and numerically established. The cutting motion is the resultant of the relative motions that are simultaneously performed between the cutting edges of the cutters and tooth flanks. The real speed is determined by three groups of parameters: of motion, constructive, of position. One presents the results of some numerical and experimental applications regarding the tooth processing on a machine of FD Cugir family.

Key words: cylindrical gears, milling head, motion parameters, main motion, real cutting speed.

1. INTRODUCTION

The diversity of gearing types, form sand sizes, precision, productivity, volume and efficiency of production, have determined the existence and utilization of a great number of machine types, processes, and tools for tooth generation. The increasing exigency regarding the kinematic accuracy, smooth running, durability and portant capacity of the gearings [6], make the flank processing to be one of the most complex kinematic surfaces generation modes.

The curved teeth in cylindrical gears represents preoccupations of many researchers presented in papers, doctoral thesis and/or patents, among them being [5].

The hypocycloidal flanks of the gears are defined [3] by two cycloid curves: involute – regarding the profile, and a segment of the closed hypocycloid loop, lengthened or shortened, as flank line. The two curves are generated kinematically through simple motions, strictly correlated by the generation kinematics.

2. KINEMATICS OF HYPOCYCLOIDAL FLANK GENERATION

The flank line $D$ (closed hypocycloid $hh_1$ – Fig. 1) is generated kinematically in the plane $[Γ_3]$ as trajectory of a point belonging to the rolling curve $R$, which rolls inside the fixed base $B$. The flank lines $D'$ are generated by transposing by rolling the plane curves $D$ on the revolution surface $Σ$ of the workpiece-gear $P$. The number of circles $R$ corresponds to the ratio $i_H = R_b / R_R$ ($R_b$ is the base radius and $R_R$ – rolling curve radius).

These are positioned equidistantly inside the base $B$ and revolute simultaneously with the angular speed $ω_R$. To each rolling curve two cutters are attached by means of a port-tool support. The cutters have principal cutting edges ($T_s$ and $T_d$) and secondary ones.

The assembly formed by the simple planetary mechanism (the base and the $i_H$ rolling curves) and the corresponding cutter couples attached to each rolling curve constitutes a cutting tool of multi-cutter ($i_H = 3, 4, 5$ or 6) milling head (CF) type [5]. The cutting tool can be adapted on the tooth milling machine FD 320A-Cugir [4]. The gearing formed from the base $B$ and the $i_H$ rolling curves $R$ and the revolution couples created in a driving disk $D_a$ with the revolution axis in $O_2$ and in which the rolling curves axes are materialized corresponding to certain specific accuracy conditions: rigidity, running without noise and vibrations, smooth motion, backlash free and modularization.

The involute profile of the flanks is also generated kinematically [1] as envelope of the successive positions of the two cutting edges $T_s$ and $T_d$ [3], which materialize in the median plane of the plane wheel $P$ and in parallel planes to this one the generating rack profile with the
reference line NN tangent to the rolling circle \( C_P \) of the gear \( P \). The reproduction of the profile is obtained by continuous division [1], in which case the revolution motions of the tool with the speeds \( \omega_R \) and \( \omega_P \) and of workpiece \( \omega_P \) are continuous. Between the three motions and their angular speeds some relations has to be established, which are imposed by the generation kinematics, one of them being of form: \( \omega_R = i_M \cdot \omega_D \).

The rolling speed \( v_r \) between the workpiece-gear (surface \( \Sigma_2 \)) and the plane \( \Pi_D \) expresses the generating speed of the involute profile. The speed \( v_r \) is from the technological point of view the speed of the tangential feed motion of the saddle \( ST \) that supports the tool.

The generation kinematics of the flanks imposes the existence of the three circular motions and linear continuous motion \( (v_r) \) supplied by four generating kinematic chains represented in Fig. 1, namely: main, tangential feed, rolling for generation of the curve \( D \), rolling for the involute profile. The trajectories of the generating motions are supplied by simple kinematic couples: shaft-bearing and saddle-guide respectively existent in the kinematic structure of the teeth processing machine.

The main kinematic chain adjusted through the change gears \( A_v/B_v \) ensures the motion of each group of cutters on hypocycloidal trajectories. Thus, the cutting edges \( T_s \) and \( T_i \) effectuate the main motion with the cutting speed tangent to the shortened hypocycloids, respectively elongated, which are flank directrices.

The feed motion with the speed \( v_r \) is obtained at the end of the tangential feed kinematic chain consisting of: \( M_E_2 - CA - C_2 \) (pos. 1) – \( L_2 - C_3 \) (1) – \( M_T1 \) and tangential saddle \( ST \).

The rolling kinematic chain for generating the directrix \( hh_1 \) in plane \( \Pi_D \) is formed by: \( CF (Da) - D_{FF} - \text{Diff} - A_{RD}/B_{RD} \) and workpiece \( P \) (joint \( O_{PS}, n_{PD} \)). Hence, a second condition of correlation of the revolution motions is fulfilled, having the form:

\[
\varepsilon_{PD} = \varphi_D \cdot \frac{i_{DH}}{z_p} \text{[degrees]},
\]

where \( \varepsilon_{PD} \) represents the revolution angle of the workpiece for continuous division and \( \varphi_D \) – revolution angle of the driving disk of the milling head \( CF \).

The rolling kinematic chain for generation of the involute profile is formed by the generating rack (line \( NN \)) attached to the plane \( \Pi_D \), and consists of: \( L_2 - A_{RG}/B_{RG} - \text{Diff} - A_{RD}/B_{RD} \) and \( P(n_p) \). The Botez mechanisms is formed by workpiece-gear and generating rack defined in the mean plane. On the basis of the closing condition of this kinematic chain the following relation is established:

\[
\varepsilon_{PR} = \varphi_D \cdot \frac{s_T \cdot i_{DH}}{2 \pi \cdot z_p \cdot R_r} \text{[rad]},
\]

where \( s_T \) represents the tangential feed of the saddle \( ST \), in mm/(rev. \( P \)); \( R_r \) – rolling radius of the workpiece, in mm; \( z_p \) – tooth number of the workpiece. From relations (1) and (2) the revolution angle \( \varepsilon_P \) of the workpiece results, in radians, corresponding to a certain angle \( \varphi_D \) in radians, expressed by the relation:

\[
\varepsilon_P = \varepsilon_{PD} + \varepsilon_{PR} = \varphi_D \cdot \frac{i_{DH}}{z_p} \left(1 + \frac{s_T}{2 \pi \cdot R_r}\right) \text{[rad]},
\]

on which basis the workpiece speed \( n_p \) is determined.

The generation motion parameters are considered constant in the cutting process. The rolling motion between workpiece and \( CF \) is continuous and motion along the flank profile is discontinuous.

3. ANALYSIS OF CUTTING MOTION

The cutting motion is the resultant of the relative motions that are acting simultaneously, between the workpiece \( P \) (Fig. 2) and the cutting edges \( T_d \) and \( T_i \) of the group of cutters \( S_1 \) and \( S_2 \) respectively, adapted on each rolling curve of the \( CF \).

The motions of the cutters and gear are achieved by the four kinematic chains and satisfy the kinematic simultaneous generation condition of the profile and flank.

Fig. 2. Relative position tool – workpiece: a – reference frames and motions; b – definition of the current point \( M_d \).
line. The definition of the trajectory of the cutting motion is necessary for the analytical and numerical determination of the forms of flank line and active angles of the cutting edges of the tools.

3.1. Components of the cutting motion

The revolution motion of the centres of each rolling curve in regard with the centre of revolution axis of $CF$ is defined through the angular parameter $\varphi_D$ (Fig. 2a).

In the reference frame of the tool $S_D (O_D X_D Y_D Z_D)$ the cutting edges $T_c$ and $T_d$ are defined by the equations:

\[
\begin{align*}
X_S &= -(R_R + k \frac{\pi \cdot m}{4}) + k \cdot u \cdot \cos \alpha_0 \\
Y_S &= -u \cdot \sin \alpha_0 \\
Z_S &= 0,
\end{align*}
\]

(4)

where $k = \pm 1$ corresponds to the cutting edges $T_c$ and $T_d$ respectively; $u$ — parameter that defines points of the cutting edges on their length; $\alpha_0$ — the angle of the reference profile.

The parametric equations of the normal hypocycloidal trajectory $H_u$ in the reference frame $S_D$ of $CF$ have the form:

\[
\begin{align*}
X_D &= (R_R - R_k) \cos \varphi_D - R_R \cos \frac{R_R - R_k}{R_R} \varphi_D \\
Y_D &= (R_R - R_k) \sin \varphi_D - R_R \sin \frac{R_R - R_k}{R_R} \varphi_D \\
Z_D &= 0,
\end{align*}
\]

(5)

where $\varphi_D$ represents the parameter of the revolution motion of $CF$. The plane $O_X Y Z_S$ which contains the cutting edges, changes continuously its position in regard with the plane $Y = 0$.

A current point $M_{s,d} \in T_{s,d}$ of instantaneous contact between workpiece and tool edge located at the radius $R_{Ms,d} = R_R \pm a$ will generate an elongated respectively shortened hypocycloidal trajectory.

The parameter $a$ defines the change of point $M_{s,d}$ considered on the cutting edge $T_c$ and $T_d$ respectively in regard with $O_S$.

Main motion speed is determined with relation:

\[
v_{as} = v_P = 2\pi \cdot n_p \cdot (R_R - R_k).
\]

(6)

where $f = 1 + \frac{R_k^2}{R_R} \cdot f \cdot \cos \frac{R_R - R_k}{R_R} \varphi_D \cdot 10^{-3}$, and $n_p$ — speed of $CF$, in rot/min. The vector of this speed is tangent to the hypocycloidal trajectory that is generated kinematically as trajectory of a point.

From relation (6) it results that the speed $v_{as}$ in m/min has a variable instantaneous direction (parameter $\varphi_D$) and size corresponding to the radius of current point $M_{s,d}$ (Fig. 2b). The generated flanks belong to the work-piece defined through the rolling circle $C_p$. The work-piece is framed in the frame $S_P (X_P O_Y Z_P)$ (Fig. 2a) named director kinematic system considered fixed for this study.

The components of the vector $v_{as}$ defined in the frame $S_P$ have the following expressions:

\[
\begin{align*}
v_{as,x_P} &= 2\pi \cdot n_p \cdot (R_R - R_k) \left( \sin \varphi_D - \frac{f}{R_R} \cdot \sin \frac{R_R - R_k}{R_R} \varphi_D \right), \\
v_{as,y_P} &= 2\pi \cdot n_p \cdot (R_R - R_k) \left( \cos \varphi_D - \frac{f}{R_R} \cdot \cos \frac{R_R - R_k}{R_R} \varphi_D \right), \\
v_{as,z_P} &= 0.
\end{align*}
\]

(7)

Tangential speed $v_P$ of the workpiece is constant and has the direction of the revolution motion and the value determined by the parameter $n_p$. The work-piece is considered fixed, so that the speed $v_P$ will be equal and of contrary direction to the speed of an equivalent motion corresponding to a current point $M_d$ (Fig. 2b) that belongs to the cutting edge $T_d$.

The components of the vector $v_P$ are determined by relations:

\[
v_{p,x_P} = (v_P - c_{vp}) \cdot a_{vp}, \quad v_{p,y_P} = (v_P - h_{vp}) \cdot b_{vp}, \quad v_{p,z_P} = (v_P - c_{vp}) \cdot c_{vp}
\]

(8)

where $v_P = 2\pi \cdot n_p \cdot R_{Md}$ and $a_{vp} = -\cos \eta$, $h_{vp} = 0$, and $c_{vp} = -\sin \eta$ represent the director cosines of the vector $v_P$

\[
\eta = \left( \gamma_p + \beta_p \right) - \varphi_P
\]

— angular parameter that defines the instantaneous position of the point $M_d$ with respect to the axis $Z_P$, $\gamma_p = 90^\circ / z_p$, $\beta_p = \varepsilon_p + \alpha_p = \alpha_p$ (as $R_{Md} < R_s$, respectively $R_{Md} > R_s$), $z_p$ — tooth number of the work-piece, and $\alpha_p = \arccos \left[ \left( \frac{R_s}{R_{Md}} \right) \cos \alpha_0 \right]$. The angular parameter $\varepsilon_p$ expresses the angular position of the work-piece, corresponding to the current point $M_d$. It has a dependent relation with the parameter $u$, namely: $u = 1 / k \left[ \left( R_k e_p + k \cdot m \cdot \pi / 4 \right) \cdot \sin \alpha_0 \right]$.

The point $M_d$ radius is calculated with the relation:

\[
R_{Md} = \left( X_P^2 + Z_P^2 \right)^{1/2},
\]

(9)

where $X_P$ and $Z_P$ are defined in (3) and represents the coordinates of the point belonging to the generated involute profile, corresponding to the rolling parameter $\varepsilon_p$. For simplifying the calculations, the mean palne $[Y_P]$ is considered, for which it corresponds $\varphi_D = 0$. After necessary computation, it results in mm:

\[
R_{Md} = \frac{R_s^2 \left( 1 + 2 \cdot k \cdot R_s \cdot \varepsilon_p \frac{m \pi}{4} \right) + \left( k \cdot u \frac{m \pi}{4} \right)^2}{\left[ 2 R_s \cdot \cos \alpha_0 - k \cdot r_p \frac{m \pi}{4} \right] \sin^2 \alpha_0}
\]

(10)

Component of the rolling motion with the speed $v_P$ is along the line $NN$ and has the direction of the revolution motion of the work-piece for rolling with speed $n_{pr}$ (Fig. 2). The projections of the vector $v_P$ on the axis of the frame $S_P$ have the expressions:
where \( n_{pr} = \left( s_f \cdot i_f / 2 \pi \cdot z_p \cdot R_s \right) \cdot n_D \), in \( \text{rot/min} \); \( s_f \) - tangential feed \( \left( s_f = s_t \cdot z_p / i_f \right) \), in \( \text{mm/(rev. P)} \); \( s_t \) - tangential feed saddle \( ST \), in \( \text{mm/(rev. CF)} \); \( \varepsilon_{pr} \) - angular parameter of the involute profile generation.

**Real cutting speed** \( v_{ras} \) is determined [2] as sum of the simultaneous motion speeds, namely:

\[
v_{ras} = \bar{v}_a + \bar{v}_p + \bar{v}_s,
\]

expressed by the relations (6, 8, and 11). The projections of the vector \( \bar{v}_{ras} \) on the axes of the frame \( S_p \) are:

\[
\begin{align*}
(v_{ras})_{xp} &= 2n_D \cdot \pi \left\{ \left( R_B - R_s \right) \sin \varphi_D - \frac{R_B - R_s}{R_R} \cdot f \cdot \sin \frac{R_B - R_R}{R_R} \varphi_D + \frac{i_H}{z_p} \left[ R_{md} \cdot \cos \eta + \frac{s_f}{2\pi} \cos \varepsilon_{pr} \right] \right\}, \\
(v_{ras})_{yp} &= 2n_D \cdot \pi \left\{ \left( R_B - R_s \right) \cos \varphi_D - \frac{R_B - R_s}{R_R} \cdot f \cdot \cos \frac{R_B - R_R}{R_R} \varphi_D \right\}, \\
(v_{ras})_{zp} &= 2n_D \cdot \pi \left\{ i_H \frac{R_{md}}{z_p} \cdot \sin \eta + \frac{s_f}{2\pi} \cdot \sin \varepsilon_{pr} \right\}.
\end{align*}
\]

The real cutting speed vector is variable concerning the value and direction. The determination of its components is necessary for defining the active geometry of the cutters edges and for analysis of the cutting process. On the basis of relations (13) the modulus of the vector \( \bar{v}_{ras} \) is determined in \( \text{m/min} \):

\[
|\bar{v}_{ras}| = \sqrt{(v_{ras})_{xp}^2 + (v_{ras})_{yp}^2 + (v_{ras})_{zp}^2} \cdot 10^{-3}.
\]

The direction of vector \( \bar{v}_{ras} \) in every point of the hypocycloidal trajectory on the length and height of generated flanks with respect to the axes of the frame \( S_p \) is determined by calculation of its director cosines [2].

**Application.** For numerical expression of the above relations it is considered the case of generation of hypocycloidal flanks by rolling, by applying the procedure presented in Fig. 3, for a cylindrical gear (workpiece) having \( z_p = 52 \) and \( m = 2.5 \text{ mm} \); milling head with 4 groups of cutters and geometric characteristics \( R_B = 240 \text{ mm} \) and \( R_R = 60 \text{ mm} \), speed \( n_D = 29.2 \text{ rpm} \); feed \( s_f = 0.83 \text{ mm/(rev. P)} \); \( \eta = 3.32 \); \( \varphi_D = 20.0 \text{ deg} \); \( k = -1 \) (for \( T_d \)) and \( \alpha_p = 20 \text{ deg} \).

On the basis of the above relations one determines:

\[
\begin{align*}
\bar{v}_{ras} &= -59.173 \text{ mm/min}, \quad \bar{v}_a = 65.2 \text{ mm/min}; \quad R_{md} = 68.93 \text{ mm}, \quad \text{components} \quad (v_{ras})_{xp} = 3.2 \text{ m/min}; \quad (v_{ras})_{yp} = 65.185 \text{ m/min} \quad \text{and} \quad (v_{ras})_{zp} = 0.037 \text{ m/min}. \quad \text{According to relation (13)} \quad \text{it results} \quad |\bar{v}_{ras}| = 65.28 \text{ m/min}. \quad \text{We ascertain that the difference between the values of the speeds} \quad \bar{v}_{ras} \quad \text{and} \quad \bar{v}_a \quad \text{is very small}, \quad \text{this being the reason} \quad \text{that in studying the cutting regime parameters one can consider only the parameter} \quad \bar{v}_a.
\end{align*}
\]

**Fig. 3.** Milling head with 4 groups of cutters.

4. CONCLUSIONS

The flank line and profile of the hypocycloidal curved teeth are generated kinematically by rolling with mobile line and continuous division. For processing a milling head \( CF \) is used, which has more couples of cutters. This tool is adapted on the tooth processing machine of FD Cugir type. The main motion has the speed \( v_{ras} \) with the biggest value of the components of the real cutting speed. The size of the cutting speed is determined by the ratio \( i_f \) and the driving disk speed. Thus, for the considered numerical values, it was established the range: \( v_{af} = 56.95 \ldots 72.1 \text{ m/min} \). Knowing the projection values of the vector \( v_{ras} \) in the workpiece frame and its modulus, one can determine the parameters that define the active geometry of the cutting edge of each cutter and the orientation of their faces and cutting edges in working position on the tooth processing machine.

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