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INCREASE PRECISION OF PARALLEL MANIPULATORS

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Abstract: *The parallel manipulators are used for many industrial applications, thanks to their better performances: high precision and rigidity, high load capacity and high ratio between payload and net weight. On the other hand, the parallel manipulators suffer of some drawbacks. A very important one is the presence of singularities inside the workspace. It is very well known that the positioning accuracy of the platform of a parallel manipulator decrease with approaching to the singular position. One of the aims of the optimum design of the parallel manipulators is to obtain better performance, regarding the positioning precision of the platform and implicit – of the end – effector, and the decrease of the driving forces. The present paper discuses in detail some aspects of the optimization problem of the parallel manipulator design, so that the positioning accuracy to be maximum.*

Key words: *kinematics analysis, parallel manipulators, manipulator design, optimum synthesis.*

1. INTRODUCTION

The parallel manipulators consist of a platform, which is connected to the base by many of independent open kinematic chains. As a rule, the platform may occupy any position in space.

Therefore, the platform of a 6-DOF parallel manipulator, so called Gough–Stewart platform, is operated by six actuators, which are included in the connection kinematic chains.

It is very well known that the positioning accuracy of the platform is decreasing with the approaching to a kinematic singular position.

An important shortcoming of the parallel manipulators is that they operated in reduced workspaces in order to avoid the singular configurations.

The design may be idealized using various mathematical models.

The techniques of idealization played a decisive role in the success of optimization process.

The goal of the optimum kinematic synthesis of the parallel manipulators is to establish the kinematic dimensions of the component elements provided so that some conditions regarding the precision of the positioning of the platform are imposed.

In the optimization problems, the number of the unknown parameters, which define the geometry of a 6-DOF parallel manipulator, is very large. In a general case, this number is 162 at which is added the variables of the driving pairs.

A lot of these parameters is constant and defines the kinematic dimensions of the component elements, and the rest are the variables of the kinematic pairs. For symmetry reasons, the number of constant parameters is much small.

For example, the actuator kinematic chains may be grouped by twos, and these pairs may be equidistant disposed around the center of the frame.

But, only in the special cases, this assumption may not be accepted.

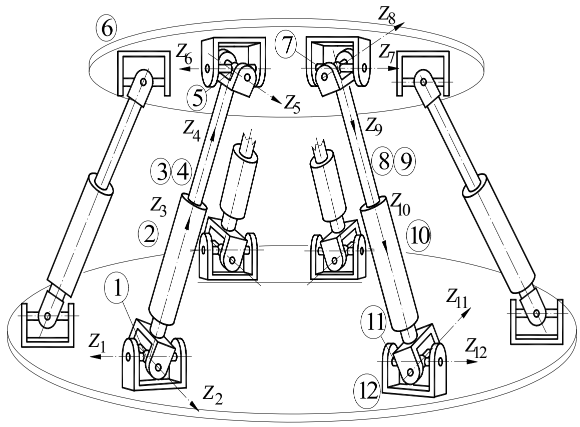

Fig. 1. Kinematic diagram of parallel manipulator.

Note that the solutions of an optimization problem depend on the application which must be performed by the manipulator [1, 3].

The design has been idealized into a mathematical model for the purpose of analysis; the techniques of idealization can play a decisive role. The mathematical model based on the Denavit – Hartenberg formalism [2], [5] may not only simplify the problem formulation, but can also yields considerable advantage in the solving of the problem.

Moreover, the use of the Denavit – Hartenberg formalism to write the analysis equations led to a very well conditioned system and the convergence of anyone numerical resolution is faster.

These equations emphasize all kinematic dimensions of the component links and facilitated the optimization procedure.

Through this methodology appears as elaborate, it is able to reduce the complexity of the optimum design problem to a manageable level.

In the following are analyzed the possibilities of the design of the 6–DOF parallel manipulators so that the positioning accuracy of the platform to be maximum.

2. ANALYSIS OF THE PARALLEL MANIPULATOR

The direct kinematic analysis of the mechanism of the parallel manipulators is generally difficult to be solved [4] allowing a great number of equations and multiple solutions. Moreover, in the neighborhood of the singularities, the computing of the direct kinematic is very hard or even impossible, because the value of the determinant of the Jacobean matrix of the analysis functions is very small or null.

The actuators must posses high-resolution displacements in order to realize a precise displacement of the platform.

The influence of the errors of the actuator displacements to the precision of the platform positioning depends with the ratios between infinitesimal displacements of the platform and the infinitesimal displacements of each actuator.

Therefore, it is necessary that the maximum value of the derivatives of the platform position parameters with respect to the actuator displacements (generalized coordinates) must be minim.

In the other words, the ratios between platform infinitesimal displacements and the infinitesimal actuators displacements must be minim.

In a singular position, a mechanism in general, a parallel manipulator in particular, gains one or more degrees of freedom instantaneously.

In other words, if a parallel robot is in a singular configuration, it loses its designated motion and working capability.

In such configurations, the parallel manipulator loses its rigidity becoming locally movable, even if the actuators are blocked.

Therefore, the derivative of the positional parameters of the platform with respect to the variables of actuate joints tend to infinity and the system gets unstable.

2.1. Direct Kinematic Analysis

Usually, the linear actuators are joined to the base by universal joints and to the platform by spherical joints.

In Fig. 1 are numbered and the fictitious links [2]. The kinematic analysis may be made very easy using the well-known Denavit-Hartenberg transformation matri-ces.

The mechanism has five independent closed loops. The matrix loop equations for analysis of the spatial mechanism of a 6-DOF parallel manipulator are:

$$\mathbf{A}_{i1}\mathbf{A}_{12}\mathbf{A}_{13}\mathbf{A}_{14}\mathbf{A}_{15}\mathbf{A}_{i6}\mathbf{A}_{i7}\mathbf{A}_{i8}\mathbf{A}_{i9}\mathbf{A}_{i,10}\mathbf{A}_{i,11}\mathbf{A}_{i,12}=\mathbf{I},\quad i=\overline{1,5}, \tag{1}$$

where: $\theta_{1j}=\theta_{11}+u_j$, $\theta_{6j}=\theta_{61}+v_j$, $j=\overline{2,5}$; u_i and v_i are considered as known.

The Denavit – Hartenberg transformation matrices $\mathbf{A}_{12}$, $\mathbf{A}_{13}$, $\mathbf{A}_{14}$ and $\mathbf{A}_{15}$ are commonly to all five loops.

The unknowns of the system (1) are: θ_{1k}, k = 1, 2, 3, 5, 6; θ_{ij}, $j=\overline{7,9}$, $i=\overline{1,5}$; θ_{ij}, j = 11, 12, $i=\overline{1,5}$.

The variables s_{14} and $s_{j,10}$, $j=\overline{1,5}$ of the driving prismatic pairs form the vector of generalized coordinates of the manipulator and are known. First subscript of the Denavit – Hartenberg matrix **A** is referred to the number of the loop and the second subscript – to the number of the matrix into the loop.

The solutions are well defined except the case when the determinant of the Jacobian matrix is very small, i.e. in the neighborhood of a dead – point. In the initial position of the mechanism of the parallel manipulator must be known the initial estimates of all pair variables.

To solve the analysis problem for a new position, the vector of generalized coordinates must be incremented by a small amount and the previous calculated unknowns are used as initial estimates for the new position.

The position of a point *P* belonging to the platform (6) is defined with respect to the frame coordinate axes system by matrix equation (2):

$$\mathbf{A}_{11}\,\mathbf{A}_{12}\,\mathbf{A}_{13}\,\mathbf{A}_{14}\,\mathbf{A}_{15}\,\mathbf{A}_{16}\,\mathbf{X}_{P7}=\mathbf{X}_{P1}, \tag{2}$$

where $\mathbf{X}_{P7}=\left\|1\ x_{P7}\ y_{P7}\ z_{P7}\right\|^{\mathrm{T}}$ is the vector of the coordinate of the point *P* with respect to the mobile system $o_7x_7y_7z_7$ attached to the platform (6) and $\mathbf{X}_{P1}=\left\|1\ X_{P1}\ Y_{P1}\ Z_{P1}\right\|^{\mathrm{T}}$ is the vector of the coordinate of the same point with respect to the fixed coordinate axes system.

The derivatives of the revolute pair variables with respect to the time are calculated as solutions of the following equations:

$$\begin{aligned}
&\frac{\partial\mathbf{A}_{i1}}{\partial\theta_{i1}}\mathbf{A}_{i1}\mathbf{A}_{12}\mathbf{A}_{13}\mathbf{A}_{14}\mathbf{A}_{15}\mathbf{A}_{i6}\mathbf{A}_{i7}\mathbf{A}_{i8}\mathbf{A}_{i9}\mathbf{A}_{i,10}\mathbf{A}_{i,11}\mathbf{A}_{i,12}\frac{\mathrm{d}\theta_{i1}}{\mathrm{d}t}+\\
&+\mathbf{A}_{i1}\frac{\partial\mathbf{A}_{12}}{\partial\theta_{12}}\mathbf{A}_{13}\mathbf{A}_{14}\mathbf{A}_{15}\mathbf{A}_{i6}\mathbf{A}_{i7}\mathbf{A}_{i8}\mathbf{A}_{i9}\mathbf{A}_{i,10}\mathbf{A}_{i,11}\mathbf{A}_{i,12}\frac{\mathrm{d}\theta_{12}}{\mathrm{d}t}+\\
&+\mathbf{A}_{i1}\mathbf{A}_{12}\mathbf{A}_{13}\frac{\partial\mathbf{A}_{14}}{\partial s_{14}}\mathbf{A}_{15}\mathbf{A}_{i6}\mathbf{A}_{i7}\mathbf{A}_{i8}\mathbf{A}_{i9}\mathbf{A}_{i,10}\mathbf{A}_{i,11}\mathbf{A}_{i,12}\frac{\mathrm{d}s_{14}}{\mathrm{d}t}+\ldots+\\
&+\mathbf{A}_{i1}\mathbf{A}_{12}\mathbf{A}_{13}\mathbf{A}_{14}\mathbf{A}_{15}\mathbf{A}_{i6}\mathbf{A}_{i7}\mathbf{A}_{i8}\mathbf{A}_{i9}\frac{\partial\mathbf{A}_{i,10}}{\partial s_{i,10}}\mathbf{A}_{i,11}\mathbf{A}_{i,12}\frac{\mathrm{d}s_{i,10}}{\mathrm{d}t}+\ldots+\\
&+\mathbf{A}_{i1}\mathbf{A}_{12}\mathbf{A}_{13}\mathbf{A}_{14}\mathbf{A}_{15}\mathbf{A}_{i6}\mathbf{A}_{i7}\mathbf{A}_{i8}\mathbf{A}_{i9}\mathbf{A}_{i,10}\mathbf{A}_{i,11}\frac{\partial\mathbf{A}_{i,12}}{\partial\theta_{i,12}}\frac{\mathrm{d}\theta_{i,12}}{\mathrm{d}t}=0
\end{aligned} \tag{3}$$

$i=\overline{1,5}$.

The components of the velocity of the point P on the fixed coordinate axes system are calculated with equation (4).

$$\begin{aligned}
&\left(\frac{\partial\mathbf{A}_{11}}{\partial\theta_{11}}\mathbf{A}_{12}\mathbf{A}_{13}\mathbf{A}_{14}\mathbf{A}_{15}\mathbf{A}_{16}\frac{\mathrm{d}\theta_{11}}{\mathrm{d}t}+\right.\\
&+\mathbf{A}_{11}\frac{\partial\mathbf{A}_{12}}{\partial\theta_{12}}\mathbf{A}_{13}\mathbf{A}_{14}\mathbf{A}_{15}\mathbf{A}_{16}\frac{\mathrm{d}\theta_{12}}{\mathrm{d}t}+\ldots+\\
&+\mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{13}\frac{\partial\mathbf{A}_{14}}{\partial s_{14}}\mathbf{A}_{15}\mathbf{A}_{16}\frac{\mathrm{d}s_{14}}{\mathrm{d}t}+\ldots+\\
&\left.+\mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{13}\mathbf{A}_{14}\mathbf{A}_{15}\frac{\partial\mathbf{A}_{16}}{\partial\theta_{16}}\frac{\mathrm{d}\theta_{16}}{\mathrm{d}t}\right)\mathbf{X}_{P7}=\frac{\mathrm{d}\mathbf{X}_{P1}}{\mathrm{d}t},
\end{aligned} \tag{4}$$

where $\dfrac{\mathrm{d}\mathbf{X}_{P1}}{\mathrm{d}t}=\left\|0\ \dfrac{\mathrm{d}X_{P1}}{\mathrm{d}t}\ \dfrac{\mathrm{d}Y_{P1}}{\mathrm{d}t}\ \dfrac{\mathrm{d}Z_{P1}}{\mathrm{d}t}\right\|^{\mathrm{T}}$.

The positions and velocities of the inputs must all be known as functions of time or some other independent parameter.

2.2. Inverse Kinematic Analysis

In the inverse kinematic analysis the equations (1) and (2) are simultaneously solved. The unknowns are:

$$\theta_{1j}, j = 1, 2, 3, 5, 6;\ \ \theta_{ij},\ j = \overline{7, 9}, 11, 12,\ i = \overline{1, 5};$$

$$s_{1,4};\ s_{i,10},\ i = \overline{1, 5},$$

as function of the coordinate X_{P1}, Y_{P1}, Z_{P1} of the point P and the elements of the **DC** orthogonal submatrix of the direction cosines, which define the orientation of the platform:

$$\mathbf{dc}(i, j) = m(i, j),\quad i = \overline{2, 4},\quad j = \overline{2, 4}, \tag{5}$$

where $\mathbf{M} = \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{13}\mathbf{A}_{14}\mathbf{A}_{15}\mathbf{A}_{16}$. The elements of the matrix $\mathbf{M}$ are denoted by $m(i, j)$.

From nine direction cosines of the **DC** submatrix, which set up the elements of the matrix **M**, only three are independents.

But, all nine values must be specified, because the relationship equations are nonlinear. If the link equations between direction cosines are solved simultaneously, this produces eight sets of six different solutions:

$$\begin{aligned}\frac{\mathrm{d}\mathbf{M}}{\mathrm{d}t} &= \frac{\partial \mathbf{A}_{11}}{\partial \theta_{11}}\mathbf{A}_{11}\mathbf{A}_{12}\ldots\mathbf{A}_{16}\frac{\mathrm{d}\theta_{11}}{\mathrm{d}t} + \\ &+ \mathbf{A}_{11}\frac{\partial \mathbf{A}_{12}}{\partial \theta_{12}}\mathbf{A}_{13}\ldots\mathbf{A}_{16}\frac{\mathrm{d}\theta_{12}}{\mathrm{d}t} + \ldots + \\ &+ \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{13}\frac{\partial \mathbf{A}_{14}}{\partial s_{14}}\mathbf{A}_{15}\mathbf{A}_{16}\frac{\mathrm{d}s_{14}}{\mathrm{d}t} + \ldots + \\ &+ \mathbf{A}_{11}\ldots\mathbf{A}_{15}\frac{\partial \mathbf{A}_{16}}{\partial \theta_{16}}\frac{\mathrm{d}\theta_{16}}{\mathrm{d}t}.\end{aligned} \tag{6}$$

To diminish the time of calculus, may be used a simplified form of the Denavit-Hartenberg transformation matrices:

$$\mathbf{A}_{ij} = \begin{Vmatrix} a_{ij}\cos\theta_{ij} & \cos\theta_{ij} & -\sin\theta_{ij}\cos\alpha_{ij} & \sin\theta_{ij}\sin\alpha_{ij} \\ a_{ij}\sin\theta_{ij} & \sin\theta_{ij} & \cos\theta_{ij}\cos\alpha_{ij} & -\cos\theta_{ij}\sin\alpha_{ij} \\ s_{ij} & 0 & \sin\alpha_{ij} & \cos\alpha_{ij} \end{Vmatrix}$$

This simplification takes into consideration that is not necessary to calculate the multiplication of 0 or 1 by any number. Obviously, the product of two simplified Denavit-Hartenberg matrices must be performed using an adequate rule. In this way, the time of calculus of the product matrices is shorten with 45 % approximately.

3. OPTIMUM SYNTHESIS OF PARALLEL MANIPULATOR

If the goal of the optimum kinematic synthesis is to obtain the minimum magnitude of the maximum driving force, the objective function will be $F = \max\{F_i,\ i = \overline{1, 6}\}$.

The magnitudes of the driving forces may be calculated by using of the virtual work principle:

$$\sum_{i=1}^{n}\overline{P}_i\overline{V}_{P_i} + \sum_{i=1}^{m}\overline{M}_i\overline{\omega}_i + \sum_{i=1}^{6}F_i\frac{\mathrm{d}q_i}{\mathrm{d}t} = 0$$

or:

$$\begin{aligned}&P_X\sum_{i=1}^{6}\frac{\partial X_P}{\partial q_i}\frac{\mathrm{d}q_i}{\mathrm{d}t} + P_Y\sum_{i=1}^{6}\frac{\partial Y_P}{\partial q_i}\frac{\mathrm{d}q_i}{\mathrm{d}t} + P_Z\sum_{i=1}^{6}\frac{\partial Z_P}{\partial q_i}\frac{\mathrm{d}q_i}{\mathrm{d}t} + \\ &+ M_\varphi\sum_{i=1}^{6}\frac{\partial \varphi}{\partial q_i}\frac{\mathrm{d}q_i}{\mathrm{d}t} + M_\psi\sum_{i=1}^{6}\frac{\partial \psi}{\partial q_i}\frac{\mathrm{d}q_i}{\mathrm{d}t} + M_\theta\sum_{i=1}^{6}\frac{\partial \theta}{\partial q_i}\frac{\mathrm{d}q_i}{\mathrm{d}t} + \\ &+ \sum_{i=1}^{6}F_i\frac{\mathrm{d}q_i}{\mathrm{d}t} = 0,\end{aligned} \tag{7}$$

where: φ, ψ and θ are the Euler angles which define the platform orientation with respect to the manipulator frame; q_i, $i = \overline{1, 6}$, are the generalized coordinates of the 6-DOF parallel manipulators: $q_1 = s_{14}$, $q_{j+1} = s_{j,10}$, $j = \overline{1, 5}$; X_P, Y_P and Z_P are the coordinate of the point of application of the resultant force $\overline{P} = P_X\overline{i} + P_Y\overline{j} + P_Z\overline{k}$ that are acting to the platform.

Because the variation of the generalized coordinate q_i are independents, the solutions of the equations (7) are:

$$\begin{aligned}F_i = &-P_X\frac{\partial X_P}{\partial q_i} - P_Y\frac{\partial Y_P}{\partial q_i} - P_Z\frac{\partial Z_P}{\partial q_i} - \\ &- M_\varphi\frac{\partial \varphi}{\partial q_i} - M_\psi\frac{\partial \psi}{\partial q_i} - M_\theta\frac{\partial \theta}{\partial q_i},\quad i = \overline{1, 6}.\end{aligned}$$

Note that the less values of the derivatives of the platform position parameters with respect to the actuators displacement, the less magnitudes of the driving forces.

Constrains. The minimization of the objective function is made in the presence of the constrains [1, 3] which limit the minimum distance between the axes of any two consecutive actuators, in order to avoid the interference of the actuators.

A constrain is take into consideration only if the common normal between the axes of two consecutive actuators is comprised between the centers O_{12} and O_{15}, and O_{i8} and $O_{i,11}$, $i = \overline{2, 5}$ respectively.

The equations of the axes of the first and second actuators (Fig. 1) are:

$$\frac{X - X_{O_{12}}}{l_1} = \frac{Y - Y_{O_{12}}}{m_1} = \frac{Z - Z_{O_{12}}}{n_1};$$

$$\frac{X - X_{O_{18}}}{l_2} = \frac{Y - Y_{O_{18}}}{m_2} = \frac{Z - Z_{O_{18}}}{n_2},$$

where:

$$l_1 = \frac{X_{O_{12}} - X_{O_{15}}}{D_1};\quad m_1 = \frac{Y_{O_{12}} - Y_{O_{15}}}{D_1};\quad n_1 = \frac{Z_{O_{12}} - Z_{O_{15}}}{D_1};$$

$$l_2 = \frac{X_{O_{18}} - X_{O_{1,11}}}{D_2};\quad m_2 = \frac{Y_{O_{18}} - Y_{O_{1,11}}}{D_2};\quad n_2 = \frac{Z_{O_{18}} - Z_{O_{1,11}}}{D_2};$$

$$D_1 = \sqrt{(X_{O_{12}} - X_{O_{15}})^2 + (Y_{O_{12}} - Y_{O_{15}})^2 + (Z_{O_{12}} - Z_{O_{15}})^2};$$

$$D_2 = \sqrt{(X_{O_{18}} - X_{O_{1,11}})^2 + (Y_{O_{18}} - Y_{O_{1,11}})^2 + (Z_{O_{18}} - Z_{O_{1,11}})^2};$$

$$\mathbf{X}_{O_{12}} = \mathbf{A}_{11}\mathbf{X}_O;\quad \mathbf{X}_{O_{15}} = \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{13}\mathbf{A}_{14}\mathbf{X}_O;$$

$$\mathbf{X}_{O_{18}} = \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{13}\mathbf{A}_{14}\mathbf{A}_{15}\mathbf{A}_{16}\mathbf{A}_{17}\mathbf{X}_O;$$

$$\mathbf{X}_{O_{1,11}} = \mathbf{A}_{11}\mathbf{A}_{12}\mathbf{A}_{13}\mathbf{A}_{14}\mathbf{A}_{15}\mathbf{A}_{16}\mathbf{A}_{17}\mathbf{A}_{18}\mathbf{A}_{19}\mathbf{A}_{1,10}\mathbf{X}_O;$$

$$\mathbf{X}_O = \| 1\ 0\ 0\ 0 \|^T; \ \mathbf{X}_{O_{12}} = \| 1\ X_{O_{12}}\ Y_{O_{12}}\ Z_{O_{12}} \|^T.$$

The distance between the axes of the first two driven kinematic chains (Fig. 1) is:

$$d = \frac{\begin{vmatrix} X_{O_{12}} - X_{O_{18}} & Y_{O_{12}} - Y_{O_{18}} & Z_{O_{12}} - Z_{O_{18}} \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{\begin{vmatrix} l_1 & m_1 \\ l_2 & m_2 \end{vmatrix}^2 + \begin{vmatrix} m_1 & n_1 \\ m_2 & n_2 \end{vmatrix}^2 + \begin{vmatrix} n_1 & l_1 \\ n_2 & l_2 \end{vmatrix}^2}} \geq 0.$$

The coordinates X_R, Y_R and Z_R of the intersection point R of the first actuator axis and the common normally are the solutions of the following equations:

$$m_1 X_R - l_1 Y_R - X_{O_{12}} m_1 + Y_{O_{12}} l_1 = 0;$$

$$n_1 Y_R - m_1 Z_R - Y_{O_{12}} n_1 + Z_{O_{12}} m_1 = 0;$$

$$\begin{vmatrix} X_R - X_{O_{12}} & Y_R - Y_{O_{12}} & Z_R - Z_{O_{12}} \\ l_1 & m_1 & n_1 \\ \begin{vmatrix} m_1 & n_1 \\ m_2 & n_2 \end{vmatrix} & \begin{vmatrix} n_1 & l_1 \\ n_2 & l_2 \end{vmatrix} & \begin{vmatrix} l_1 & m_1 \\ l_2 & m_2 \end{vmatrix} \end{vmatrix} = 0.$$

The coordinates X_Q, Y_Q and Z_Q of the intersection point Q of the second actuator axis and the common normally are the solutions of the following equations:

$$m_2 X_Q - l_2 Y_Q - X_{O_{18}} m_2 + Y_{O_{18}} l_2 = 0;$$

$$n_2 Y_Q - m_2 Z_Q - Y_{O_{18}} n_2 + Z_{O_{18}} m_2 = 0;$$

$$\begin{vmatrix} X_Q - X_{O_{18}} & Y_Q - Y_{O_{18}} & Z_Q - Z_{O_{18}} \\ l_2 & m_2 & n_2 \\ \begin{vmatrix} m_1 & n_1 \\ m_2 & n_2 \end{vmatrix} & \begin{vmatrix} n_1 & l_1 \\ n_2 & l_2 \end{vmatrix} & \begin{vmatrix} l_1 & m_1 \\ l_2 & m_2 \end{vmatrix} \end{vmatrix} = 0.$$

The constrains are:

$$X_{O_{12}} \leq X_P \leq X_{O_{15}};\ Y_{O_{12}} \leq Y_P \leq Y_{O_{15}};\ Z_{O_{12}} \leq Z_P \leq Z_{O_{15}};$$

$$X_{O_{18}} \leq X_Q \leq X_{O_{1,11}};\ Y_{O_{18}} \leq Y_Q \leq Y_{O_{1,11}};\ Z_{O_{18}} \leq Z_Q \leq Z_{O_{1,11}}.$$

4. EXAMPLE

Let us consider a 6-DOF parallel manipulator with following dimensions: $\alpha_{14} = \alpha_{15} = \alpha_{i1} = \alpha_{i8} = \alpha_{i,10} = \pi/2$, $\alpha_{12} = \alpha_{i7} = \alpha_{i,11} = -\pi/2$, $\alpha_{i6} = -\pi\, i/2$, $\alpha_{i,12} = \pi\, i/2$, $i = \overline{1,5}$; $s_{i1} = 1.$, $s_{i6} = 0.5$, $s_{i7} = -0.5$, $s_{i,12} = -1.$, $i = \overline{1,5}$; $s_{14} = q_1$, $s_{i,10} = q_{i+1}$, $i = \overline{1,5}$.

Has been considered three design variables, namely $a_{1,12}$, $= a_{3,12} = a_{5,12}$, α_{16} and $\alpha_{1,12}$.

The angles between axes of the revolute pairs, which are adjacent to the manipulator frame, are: $\alpha_{36} = \alpha_{16} - 2\pi/3$, $\alpha_{3,12} = \alpha_{1,12} + 2\pi/3$, $\alpha_{56} = \alpha_{16} + 2\pi/3$, $\alpha_{5,12} = \alpha_{1,12} + 4\pi/3$.

The initial values of the design variables has been: $a_{1,12} = a_{3,12} = a_{5,12} = 0.1$, $\alpha_{16} = -\pi/9$, $\alpha_{1,12} = 10\pi/18$.

As a result of the minimization process of the maximum magnitude of the position parameters derivatives of the manipulator platform, with respect to the actuator displacements, were obtained: $a_{1,12} = a_{3,12} = a_{5,12} = 0.0547014$, $\alpha_{16} = -0.722398$, $\alpha_{1,12} = 0.804420$.

The objective function has been diminished from 0.9645 to 0.838561.

The q_i, $i = \overline{1,6}$, are the generalized coordinates, *i.e.* the driving variables and has been values: $q_1 = 2.2$, $q_2 = -2.3$, $q_3 = -2.4$, $q_4 = -2.5$, $q_5 = -2.6$, $q_6 = -2.7$. All the other ones dimensions a, α and s are zero.

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