Proceedings in MANUFACTURING SYSTEMS

Proceedings in Manufacturing Systems, Volume 20, Issue 3, 2025, pp. 85-90 ISSN 2067-9238

MATHEMATICAL MODELING OF SUSTAINABLE PRODUCTION PROCESS

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Abstract: *The sustainability of the manufacturing processes requires new concepts regarding the choice of manufacturing technologies based on performance criteria, economic and environmental criteria, as well as legislative limitations. The research presented in this paper is concluded with a comprehensive theoretical formulation with practical applicability, which justifies the decisions made as for the sustainability of manufacturing systems. Thus, the paper contributes to the implementation of more energy-efficient technologies and the adoption of circular solutions (waste reduction, recycling, reuse, saving material and energy resources, energy conversions etc.). Using mathematical modeling, a multi-criteria equation is formulated with a conveniently chosen system of criteria, to assess the sustainability of manufacturing processes. This is meant to create a general mathematical model, capable of incorporating the accepted criteria, in the context of imposed technical conditions, allowing access to advantageous financing within the green transition.*

Key words: *sustainability, mathematical model, circular solutions, multi-criteria equation.*

1. INTRODUCTION

The green transition to sustainable manufacturing processes should not be a bottleneck for production and sales, but an opportunity. However, without intelligent and proportional governmental support adapted to different dimensions of the economy, the transition could accentuate imbalances and generate resistance.

Currently, both investors and banks are beginning to evaluate companies and production processes from the perspective of compatibility with a climate-neutral economy. Most of the time, this evaluation has purely economic and financial projection reasons, strictly related to the capacity of the company to operate in the future, at least up to the time horizon necessary for recovering the initial investment.

In this context, the technological process used by an economic operator must be evaluated in terms of sustainability, according to the following minimum criteria: energy consumption, total costs and compliance with environmental standards. Apart from these criteria, other dependent or secondary criteria can be also considered, such as: use of material resources, working conditions etc. [1].

The main questions that this research paper intends to answer are the following:

- how can the elements/information that define the technological process sustainability be identified?
- can a production process that meets environmental requirements be defined through a mathematical model?

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2. REVIEW OF TECHNOLOGIES USED IN THE MANUFACTURING PROCESS

At the present moment, many economic agents reconsider their core activity, starting from the challenges or opportunities of the green transition.

The main steps are:

- decarbonizing the value chain (reorganizing or redefining the operations in order to reduce the carbon footprint, from production to distribution and consumption);
- utilizing the circular economy (reuse, recycling of components etc.);
- strategic diversification (decision to leave the polluting sectors, e.g. outsourced maintenance etc.);
- reducing resource consumption (like „as-a-service" type partnerships, respectively mobility, equipment).

To ensure operational efficiency, the economic agent adopts a rethinking of production processes and flows, using a series of measures:

- energy streamlining by choosing a sustainable technological process, modernization of equipment, automating and digitizing the production lines in order to reduce consumption etc.;
- climate risk management, by integrating physical risks (flood, heat waves) into the processes of activity planning and continuity;
- selection of suppliers by requiring them to declare their carbon print and adopting ESG practices.

As part of the rewriting of manufacturing processes, financial and investment measures are also taken in the transition to a green/sustainable economy:

- compliance costs – adapting to the new regulations (e.g. EU taxonomy, CSRD, TCFD reporting) which involve additional costs;

- assets reevaluation – „brown" assets become stranded assets, as a result of depreciation/devaluation and, consequently, of the decrease in economic viability.

It can be said that the sustainability of a manufacturing process is defined as its complex technical characteristic expressed by the extent to which it meets the technical -economic and functional requirements in an imposed technological and legislative environment [2].

But the transition to a sustainable manufacturing process requires more than managerial will and ambitious internal regulations. A capital quickly and efficiently channeled toward activities that support decarbonization, energy efficiency, circular economy and climate change adaptation is needed.

3. BEHAVIORAL REACTIONS OF INDUSTRIAL PRODUCERS

An important economic approach that manufacturers are currently facing is that of the cost of sustainability of a manufacturing process, which integrates the quality and reliability of the products demanded on the market, Fig 1. In this sense, the sustainability of a production route must not limit the issue raised by ensuring the quality of the final product, namely:

- the level of quality should allow for the achievement of high benefits/profits;
- increasing profit through quality improvement.

The analysis of these issues, in the legislative context of the transition to a green economy, requires the use of an increased number of activities, which can be approached gradually, step by step, constituting the sustainability bridge towards a sustainable production process. These activities encompass the material and the binder that gives solidity to the sustainability bridge, namely:

- design,
- manufacturing planning,
- procurement,
- manufacturing process using sustainable, technologies,
- control,
- marketing,
- service,
- waste recycling.

A sustainable economic system for the production of goods has the following main elements in its structure:

- research activity;
- design activity;
- production activity;
- marketing activity.

and the following parameters:

- input;
- output.

The input parameters of an economic system of goods are justified by the performances imposed on a product/group of products (their technical parameters, technical and economic indicators). The output parameters are defined by the performances of the product produced. Thus, industrial producers are increasingly responding, but in different ways, to the pressures of green transition and climate change.

Those that integrate sustainability as part of their business strategy - not as an alignment and compliance reaction - are in a better position to manage risks, to access opportunities and to gain market trust. From this perspective, the green transition is a test of adaptability, vision and collaboration across the economy. It is based on the capacity to assess and manage the climate risks in business plans, to access green financing solutions and, respectively, to collaborate with partners with sustainable business models [3].

Studies show that large companies have a better financial position and are leaders in ESG projects and sustainable approaches, but they are also under greater pressure from the financial investors. In the case of SMEs, the following differentiation appears as a result of using governance information:

- the innovative SMEs that use market information through agility, but do not always have financial resources;
- the conservative SMEs that experience difficulties in adapting to changes and want the support of public resources or large companies.

As the green transition takes shape, it becomes clear that adaptation is not enough. Strategic anticipation and collaboration between the public sector and economic agents are needed to achieve a resilient, equitable and sustainable economic model, as shown in Table 1.

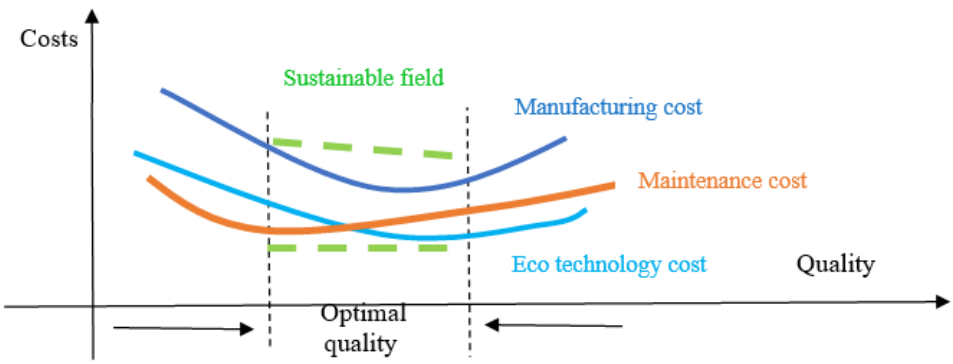

Fig. 1. Sustainable field.

Table 1

Opportunities and vulnerabilities for production fields affected by the green transition

	Opportunities	Vulnerabilities
Energy and utilities	Investments in renewable energy (solar energy, wind, green hydrogen). Development of smart grids and solutions of energy storage. Increasing demand for energy efficiency services.	Direct exposure to emission reduction policies (carbon taxes, fossil fuel phase-out). Risk of stranded assets in outdated or transition incompatible infrastructure.
Productions processes	Low–emission technologies. Carbon capture and storage (CCS) solutions. Rethinking production processes for efficiency and circularity.	High-emission technologies, exposed to strict regulations and transition costs. Decrease in the demand for products with high carbon footprint.

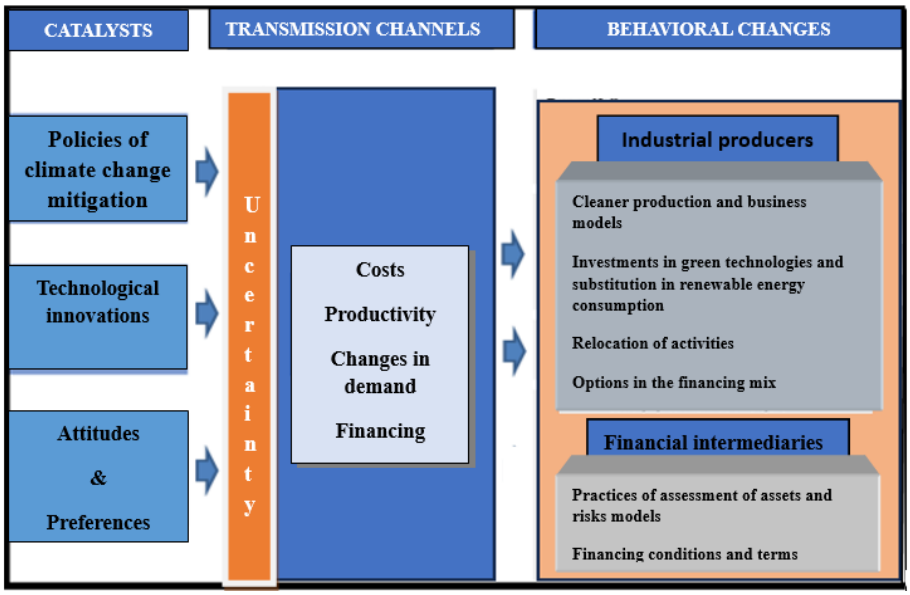

Fig. 2. Adapting green instrumentation to new structural economic conditions [2].

Transition is not just a technological or financial transformation. It is a change in the approach to production processes, priorities and mentality [3, 4].

In this endeavor, industrial producers not only have challenges to overcome, but also the real opportunity to co-create a more responsible and viable economy for future generations, Fig. 2.

4. MATHEMATICAL MODEL OF A MANUFACTURING PROCESS SUSTAINABILITY

The production activity uses in the development of technological processes directly or indirectly components or elements of various systems in society and the environment. An economic entity has a direct connection with the internal natural environment, composed of land, water source, light, heat, landforms, etc. and with the external environment of the subsoil, air, neighborhoods, society.

In order to highlight the need for mathematical modeling of a sustainable manufacturing process, it is necessary to define the eco domain and locate the system of production activities in the general system. The essential components of economic activity can be systematized by placing them in the system of the natural environment, Fig. 3. One can mark the influences coming from outside, respectively from the natural environment and society. These influences marked with +Δ (increasing) −Δ (tempering, decreasing) come from the environment and society as a result of the role played, respectively positive (+Δ), the provision of products and services, negative (−Δ) as a result of pollution, social disorder [3].

Economic activity using sustainable technological processes must manage the three concepts: production, consumption and reuse/recycling. At the same time, a

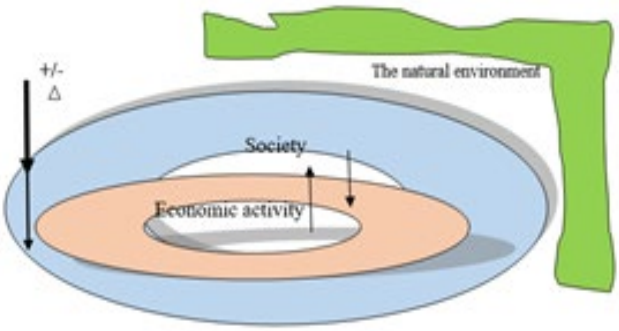

Fig. 3. Framing economic activity in the natural environment.

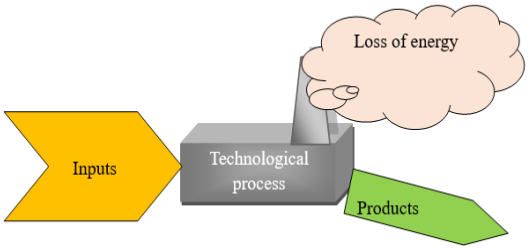

Fig. 4. The principle of entropic transformation indicator of sustainability.

The mathematical modeling of sustainable manufacturing processes must be based on the Law of Entropy [1], which defines the transformations that take place in open systems, specifying that they are carried out with losses. One can synthesize this information that will be the basis for modeling a sustainable economic process, the theory of obtaining value in the economy, Fig. 4. According to the II principle of entropy, the useful outputs from a system are always sub-unitary.

To define the elements that can form the basis for modeling a technological process, we must identify the components of a transformation process at time t_1.

These components are: *Pn, Ps, Pes.* Following the transformation process (technological process) we obtain a relationship that takes into account the inputs(energy, raw materials, etc.), expressed by the relationship:

$$Pn + Ps + Pes = Pep + E^{+}d, \qquad (1)$$

where: *Pn* – natural potential (raw materials, utilities), *Ps* – social potential, *Pes* – economic potential (assets, technological processes, etc.), *Pep* – product obtained, $E^{+}d$ – entropic losses, waste (Fig. 5). The product *Pep* contains components of information (*i*) contained in the product (s) with the help of energy (*e*), $\xrightarrow{i \quad e \quad s}$ [3].

Analyzing a manufacturing process through the lens of inputs/outputs highlights the quality of transformation and preservation of potential in the product in the three dimensions: quantity, quality, durability of qualitative properties and technological losses (released into the environment and to society without protection) [4].

Sustainable technological processes generate technological actions to minimize technological losses in the environment, in order to maintain the potential preserved in products, otherwise part of this potential will be diminished by paying fines, contributions to the restoration of the affected environment, in accordance with the legislation environment.

The use of mathematical modeling in a production process will allow for a decision to be made that will maintain the technological process in the sustainable

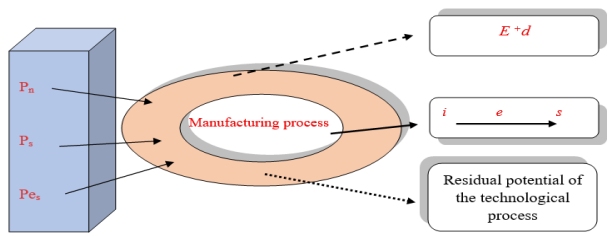

Fig. 5. Components of a manufacturing process.

range. At the same time, the sustainability of a manufacturing process can also be defined as a complex technical characteristic of it, expressed by the extent to which it satisfies the technical-economic and functional requirements in an imposed technological and legislative environment [5].

A manufacturing process aims at the partial or final making of a product at a workplace. For each manufacturing process, certain independent input variables are defined and denoted by X_i, i =1, 2, …, k. Also, there are defined certain dependent output variables denoted by Y_e, e =1, 2, …, j. There are various correlations between the input variables and the output ones, defined as process functions. The definition of the correlation functions F_{PF} is established according to the intended criterion; they can be theoretical functions, deduced on the basis of some physical factors or based on experimental results [6]. They are in the form below:

$$Y_e = F_{PF}(X_1, X_2 \ldots\ldots X_k). \qquad (2)$$

Fig. 6. Variables of a manufacturing process

For each manufacturing process (MP), (Fig.6), a global sustainability index IS – similar to the technical level of the equipment – can be defined, using the Von Neumann-Morgenstern utility principle in the Cobb-Douglas type production functions:

$$IS_j = F_{PFj}(C_{S1}, C_{S2}, \ldots\ldots C_{Sj}). \qquad (3)$$

In order to formally define the global sustainability index (*IS*), we will assimilate the technical characteristics of a machine with the sustainability assessment criteria of a manufacturing process *(CS_j)* (energy consumed, total costs, environmental impact, level of materials recycling, working conditions, etc.) [6]. It can be noticed that, in order to increase the global sustainability index, some criteria must have as high values as possible (degree of material recycling, working conditions etc. where j = 1, 2, …, f), and other criteria must have as low values as possible (energy consumed, total costs, environmental impact etc. where j = g, ..., i). The global sustainability index (*IS*) proves its practical utility when it is used to analyze two or more manufacturing processes to substantiate the decision of choosing the optimal variant in terms of sustainability [7].

The form of the multi-criteria function of the sustainability index on the basis of which it is possible to justify a decision regarding the choice or improvement of a manufacturing process a over one process b is:

$$IS_{jab} = \Pi_1^f \left({IS_{ja}}/{IS_{jb}} \right)^{\alpha_j} \times \Pi_g^i \left({IS_{jb}}/{IS_{ja}} \right)^{\alpha_j}, \qquad (4)$$

where: j = 1, 2, …, f for the criteria that must be ascending and j = g, ..., i, for the criteria that must be descending, and α_j the influence weight of the criterion j on IS_j.

In this way, by reporting the relative indices $\left(IS_{ja}/IS_{jb}\right)^{\alpha_j}$, dimensionless factors are obtained.

The exponent α_j can take values between 0 and 1. For the value 0, it turns out that the respective criterion is not important, while for the value 1, the respective criterion is the only one that matters. It is recommended to exclude the two situations. In the general variant, the following relationship must be respected:

$$\sum_1^i \alpha_j = 1. \quad (5)$$

The *consumed energy* criterion C_{EC} represents the capacity of the manufacturing process to make the same part with the lowest energy consumption:

$$IS_{EC} = 1/C_{EC}. \quad (6)$$

The *manufacturing costs* criterion C_F is the capacity of the manufacturing process to produce the same product at the lowest possible cost. This criterion can be quantified by estimating operation costs, as follows:

$$IS_{CF} = 1/C_F. \quad (7)$$

The *environmental impact* criterion C_M is the capacity of the manufacturing process to have the smallest possible impact on the environment:

$$IS_{CM} = 1/C_M. \quad (8)$$

The criterion of the C_{RM} *recycling degree* represents the percentage of recycling possibility of the material(s) from which the part is machined, after the end of the product lifespan:

$$IS_{RM} = C_{RM}. \quad (9)$$

5. CASE STUDY

A manufacturing/maintenance process for a part obtained after welding two semi-finished products is taken into consideration. The material of the two components is a VCrW85 STAS 3611-88 alloy steel which can be welded by the WIG (TIG) technique using an electric arc or by friction welding with a rotating electrode (process location – SC SOPMET SA), Fig. 7.

In the first case, namely the use of TIG deposition technique, the electric arc is formed between the non-consumable W electrode and the workpiece connected to a welding equipment. The filler metal rod (alloy) melts together with the workpiece material under a flow of shielding Ar (gas necessary for the protection of the arc and welding area). For thick parts, preheating to values ranging from 150 ºC to 200 ºC is required to prevent cracks [8].

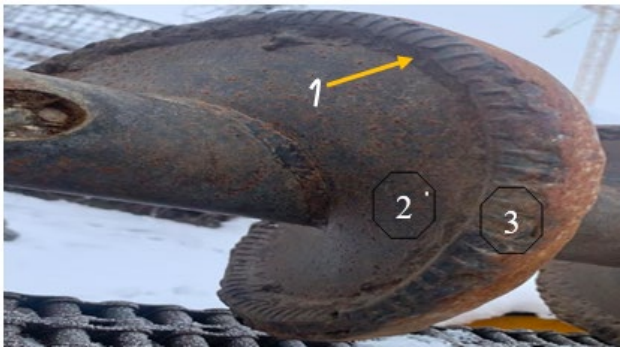

Fig. 7. Component of drilling equipment: 1- friction welding, 2- semi-finished, 3- semi-finished.

The welding process lasts 14 minutes. The technical inspection identifies an average of 2 defects per 100 parts, such as oxide inclusions from the filler metal and weld porosity, if the shielding gas Ar has low purity. The total machining cost is 200 Euro. To increase the sustainability of the machining process, welding with a rotating electrode is recommended. In this variant, a product is obtained in 11 minutes and the cost is reduced to 180 Euro. The quality of the products increases, recording one defect per 1000 products. [9].

Three criteria are used for the comparative analysis of the sustainability of the two manufacturing processes: machining time (directly proportional to the energy consumed), total cost and the quality of the parts. The following values of the priority exponents can be established for the three criteria, according to the interests of the manufacturer: $\alpha_1 = 0.4$; $\alpha_2 = 0.3$; $\alpha_3 = 0.3$.

Compared with the initial variant (welding using the TIG technique) where the value is equal to 1, in the second variant (welding with a rotating electrode), the global sustainability index will be:

$$IS = \left(11/14\right)^{0.4} \times \left(900/1000\right)^{0.3} \times \left(\frac{2/100}{1/1000}\right)^{0.3} = 2.16 \quad (10)$$

This means that by opting for a technological process that does not require expensive materials (such as W electrode and Ar) and a special welding equipment, working time and costs were reduced, while quality remained unchanged. In this case, after establishing the values of the priority exponents, the sustainability index increased more than twice from 1 to 2.16.

At the same time, analyzing the information taken from the technical studies addressed and the specialized literature, respectively information about temperature values in the technological process of fusion welding and friction welding, resulting roughness, weld seam hardness, we can make an assessment of the sustainable performance in compliance with current European standards and legislation [10].

Table 2 lists the maximum values of the indicators established for analysis, in common welding technological processes and in the analyzed friction welding technique (for reconditioning of mold active surface).

Using these values of the indices determined as basis of the research on the technological process sustainability in the reconditioning of complex molds active surfaces, we can identify and prioritize the welding technological processes by means of a Pareto chart, Fig. 8.

Current welding technologies focus on maintaining quality and reducing environmental impact, due to high polluting taxes. The analyzed indicators allow the adaptation of an efficient welding technological process, which uses minimal energy, has low carbon emissions and a favorable impact on the environment [11].

Table 2

Maximum values of the analyzed indices

	Technology / Maximum values	Temperature (°C)	Roughness (µm)	Weld seam Hardness (HRC)	Emission
1	Fusion welding	1850	12.5	60	Yes
2	Friction welding	760	7.1	65	Low quantity

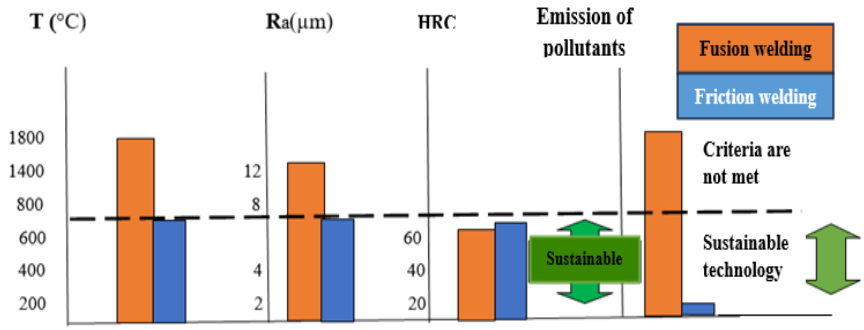

Fig. 8. Hierarchy of welding technological processes.

6. CONCLUSIONS

Sustainability is not a quantitative characteristic and for this reason it is often addressed of in general terms. As for the manufacturing processes, the method proposed by the authors entails taking into consideration a basic process (with global sustainability index equal to 1) while the other variants are compared with the basic process. Depending on the results obtained (values greater or smaller than 1 for the global sustainability index), the processes are more sustainable or less sustainable.

Thus, the integration of the economic (cost of technological process) and environmental (pollution tax) aspects will help the economic agent in maintaining the competitiveness and, implicitly, a sustainable economy, considering the distribution of pollution taxes,

The research conducted in this paper highlights the importance of establishing a global sustainability index, which makes it possible to choose the most appropriate technological process under given conditions, contributing to a green and orderly transition that maintains competitiveness, jobs and economic cohesion.

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