Proceedings in MANUFACTURING SYSTEMS
Proceedings in Manufacturing Systems, Volume 21, Issue 2, 2026, pp.61-66 ISSN 2067-9238

POSSIBILITIES FOR TURNING HYPERBOLOIDAL SURFACES

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https://doi.org/10.5281/zenodo.21861535

Abstract: *In the paper, the problem of obtaining a single-sheet hyperboloidal surface by turning was addressed. Hyperboloidal surfaces are encountered, for example, in industrial practice, in the case of rollers used for straightening cylindrical bar-shaped workpieces, and workpieces from which the gear wheels that are components of a hyperboloidal gear are to be obtained. At present, it is possible to manufacture hyperboloidal surfaces on numerically controlled machine tools, in which the cutting tools move along hyperbolas, by using numerically programmed trajectories. The fact that single-sheet hyperboloidal surfaces belong to the category of ruled surfaces also led to the idea of using machining processes in which the tools move along rectilinear trajectories, while the workpieces perform rotational movements. In this way, processes for obtaining single-sheet hyperboloidal surfaces have been developed by using wire electrical discharge machining or milling. In the paper, the idea of obtaining hyperboloidal surfaces by turning, through the use of a specialized device, is developed. The conditions under which such a device can operate and the limits of its use have been highlighted.*

Keywords: *single-sheet hyperboloidal surfaces, turning machining, device adaptable to a universal lathe, functional angles, limits of use.*

1. INTRODUCTION [1]

Hyperboloidal surfaces belong to the category of *quadric surfaces* and are of interest both from a geometric and a technological point of view. The geometric characteristics of hyperboloids, including one-sheeted hyperboloids, have been extensively described in the specialized mathematical literature [1, 2].

The specialized literature shows that hyperboloidal surfaces can be obtained by several mechanical machining processes, the most important of which are turning, milling, grinding, and wire electrical discharge machining [3, 4, 5]. For hyperboloidal surfaces of revolution, turning and milling are the most accessible solutions, as they allow the surface to be generated by combining the relative movements between the workpiece and the tool and by using universal machine tools [4]. In the case of grinding, solutions based on the use of abrasive discs with a hyperboloidal active surface have been proposed, and high-precision finishing methods have also been considered for special applications [5].

The results obtained by other researchers show that the studies carried out to date have focused mainly on the geometric generation of the surface, on the machining kinematics, and on the influence of working parameters on surface roughness. For example, in the case of turning the single-sheet hyperboloid of revolution, it was found that surface roughness is significantly influenced by feed rate, cutting speed, and tool tip radius [5].

From the analysis of the consulted works, it can be observed that the majority of research is oriented towards the geometric description of the surface and towards the kinematic conditions of generation [4, 5]. On the other hand, issues relating to the direct comparison of several machining processes applied to the same type of hyperboloidal surfaces have been less studied, as have aspects relating to form accuracy, roughness, productivity, and tool wear [4]. Furthermore, there are relatively few works oriented towards the development of simple solutions for obtaining hyperboloidal surfaces that are applicable to conventional machine tools.

In this context, the research whose results are presented in this paper had as its objective the study of the possibilities of obtaining hyperboloidal surfaces by mechanical machining processes, with emphasis on establishing the geometric and technological conditions necessary for their generation. The aim was to highlight the link between the theoretical form of the surface and the kinematics of the chosen machining process. The following chapters will present the theoretical elements relating to the generation of hyperboloidal surfaces, the processes by which they can be obtained, and a solution

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for a device adaptable to a universal lathe that could be used for obtaining hyperboloidal surfaces by turning.

2. MATHEMATICAL MODELLING OF HYPERBOLOIDAL SURFACES

In analytical geometry, a hyperbola is a plane curve belonging to the class of conic sections, which can consist of two open branches or a single branch. In canonical form, for the case in which the transverse axis coincides with the *Ox* axis, the equation of the hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \tag{1}$$

while if the transverse axis coincides with the *Oy* axis, the equation becomes:

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1, \tag{2}$$

where a is the transverse semi-axis, b is the conjugate semi-axis, and x, y are the coordinates of a point on the curve.

From a mathematical point of view, hyperbolic bodies are defined by second-order algebraic equations in three-dimensional space [1, 2]. In canonical form, the single-sheet hyperboloid is described by the relation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \tag{3}$$

while the two-sheet hyperboloid is described by the equation

$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \tag{4}$$

where c is the semi-axis along the *Oz* axis, and z is the third spatial coordinate.

In the particular case where $a = b$, the hyperboloid of revolution is obtained, a surface frequently encountered in technical applications [3].

Of the two forms of hyperboloidal surfaces, the single-sheet hyperboloid is of the greatest technological interest, as it is a continuous surface and can be generated, under certain conditions, by means of rectilinear generators [1, 3]. This property explains the use of hyperboloidal surfaces in the case of cooling towers, certain ruled structures, and other spatial structures, that is, when the aim is to ensure high rigidity and low material consumption [3]. In mechanical engineering, hyperboloidal surfaces are approached due to the complexity of their form, although the difficulties posed by their machining process are also highlighted [4, 5].

For the single-sheet hyperboloid of revolution, the equation can also be written in the form:

$$\frac{x^2+y^2}{a^2} - \frac{z^2}{c^2} = 1. \tag{5}$$

Relation (5) is useful in the analysis of the meridian profile and in establishing the conditions for surface generation [4]. This mathematical relationship is important from a technological point of view, as it allows the mathematical model of the surface to be correlated with the tool trajectory during machining.

From a practical point of view, hyperboloidal surfaces can be generated in two main ways. The first possibility consists of rotating a hyperbola around an axis. For example, if the hyperbola in the *xOz* plane is considered, whose equation in the plane is:

$$\frac{x^2}{a^2} - \frac{z^2}{c^2} = 1, \tag{6}$$

by rotating it around the *Oz* axis, the single-sheet hyperboloid of revolution described by equation (5) is obtained. If the same hyperbola is rotated around the *Ox* axis, the two-sheet hyperboloid of revolution is obtained, whose equation can also be written in the form:

$$\frac{x^2}{a^2} - \frac{y^2+z^2}{c^2} = 1. \tag{7}$$

A second possibility for generating a hyperboloidal surface is specific to the single-sheet hyperboloid, which is a doubly ruled surface. In this case, the surface can be obtained by rotating a straight line around an axis with respect to which the line is neither parallel nor intersecting [2, 3]. If a straight line described parametrically by the relations is considered:

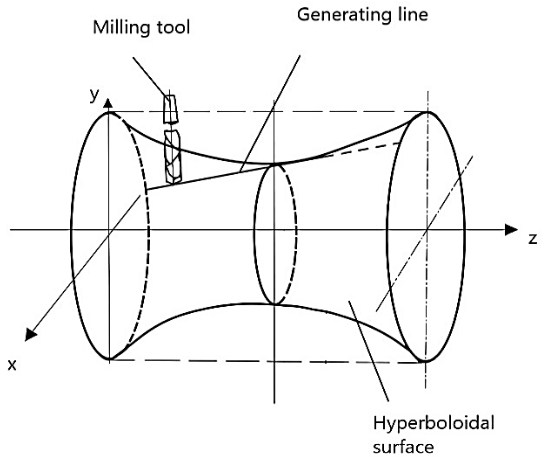

Fig. 1. Generation of a hyperboloidal surface by rotating a rectilinear generator, for example, by using milling with an end mill (adapted from [3]).

$$\begin{cases} x = a, \\ y = bt, \\ z = ct, \end{cases} \tag{8}$$

with $t \in R$, and this straight line is rotated around the *Oz* axis, a parametric surface is obtained whose points have the coordinates given by the system of equations (9):

$$\begin{cases} x = a \cdot cos\,\varphi - bt \cdot sin\,\varphi, \\ y = a \cdot sin\,\varphi + \ bt \cdot cos\,\varphi, \\ \qquad z = ct, \end{cases} \tag{9}$$

where $t \in R$ și $\varphi \in [0, 2\pi)$.

By eliminating the parameter *t*, a relation is obtained that also describes a single-sheet hyperboloid of revolution, having the same general form as equation (3), even if the geometric parameters resulting from this generation may be denoted differently.

Both generation methods are of geometric as well as technological importance. Generation by rotation of a hyperbola highlights the geometric origin of the surface, while generation by rotation of a straight line explains the ruled surface property and the practical interest shown in the single-sheet hyperboloid. In the specialized literature, it has been shown that this surface can be obtained by cutting processes, particularly by turning and milling, by taking into account the equation of the surface and the kinematic conditions of generation [4].

3. OBTAINING HYPERBOLOIDAL SURFACES THROUGH TURNING

From the specialized literature, it follows that hyperboloidal surfaces, particularly single-sheet surfaces of revolution, can be obtained by turning, milling, and, under certain conditions, by grinding. It has thus been established that a hyperboloidal surface of revolution can be obtained by cutting, using turning and milling as basic processes, based on an appropriate kinematics for generating the hyperbolic profile [3]. In the case of turning, the process is suitable for hyperboloidal surfaces of revolution, since the workpiece performs the main rotational movement while the tool is moved so that the resulting meridian profile corresponds to a hyperbola.

The same method for obtaining hyperboloidal surfaces was addressed by Rădulescu et al. [4], who experimentally analyzed the quality of the surface obtained by turning and highlighted the influence of working parameters on roughness. From this research, it follows that turning represents one of the most accessible methods for generating hyperboloidal surfaces of revolution. The authors show that milling can be applied by defining an appropriate tool trajectory, such that the points successively generated gradually materialize the desired hyperboloidal surface. In this respect, milling offers greater flexibility in the choice of kinematics, being particularly useful when surface generation is carried out on numerically controlled equipment or with adapted devices. In Figure 1 [3], the scheme for producing a hyperboloidal surface by milling is shown, while Figure 2 [3] presents the scheme for producing a hyperboloidal surface by turning can be noticed.

4. CONSIDERATIONS REGARDING THE OBTAINING OF HYPERBOLOIDAL SURFACES ON LATHES USING CUTTING TOOLS

Obtaining hyperboloidal surfaces on lathes requires the establishment of a rigorous link between the theoretical geometry of the surface and the kinematics of the generation of the tool tip. In the case of hyperboloidal surfaces of revolution, the resulting form depends on the way in which the trajectory of the tool tip is correlated with the rotational movement of the workpiece. In this context, three main solutions can be considered on lathes: machining on CNC lathes, machining with manual feed of the tool holder, and machining with mechanical feed.

In the case of CNC lathes, the hyperboloidal surface is obtained by programming the trajectory of the tool tip such that the generated meridian profile corresponds to the theoretical hyperbolic curve. The workpiece performs the principal rotational movement, while the tool is controllably displaced along directions consistent with the machine tool axes, in accordance with the numerical program. From a technological point of view, this is the most rigorous solution, as it allows precise control of the

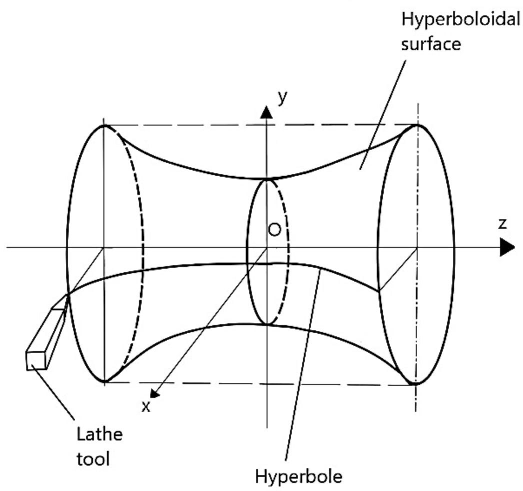

Fig. 2. Generation of a hyperboloidal surface by rotating a generator in the form of a hyperbola, for example by turning on lathes (adapted from [3]).

tool tip position along the entire length of the surface. The main advantages of the method are high accuracy, good repeatability, great flexibility with respect to the geometric dimensions of the workpiece, and the possibility of integration into an automated manufacturing process. Furthermore, modifying the form or dimensions of the surface is achieved relatively easily by modifying the program. The main disadvantages are the higher cost of the equipment, the need to develop and verify the CNC program, and the dependence on the performance of the numerical control system. Consequently, this solution is particularly recommended when the aim is to ensure high accuracy, good repeatability, and high productivity.

A second solution consists of lowering the position of the tool tip below the rotation axis of the workpiece and displacing it along a straight line distinct from this axis, which it neither intersects nor has the same direction as. As a result of the rotation of the workpiece, this movement leads to the generation of a hyperboloidal surface. From a kinematic point of view, the method is similar to that of turning conical surfaces by rotating the tool post guideway, with the difference that, in the present case, the direction of displacement of the tool tip must be chosen to satisfy the geometric condition specific to the hyperboloid.

The advantages of the method are its constructive simplicity, low cost, and the possibility of application on universal lathes, without major modifications to the machine tool. The method can be used particularly in experimental research, in the case of one-off workpieces or hyperboloidal surfaces of relatively short length. The disadvantages are, however, significant: manual feed leads to low productivity, the accuracy of the surfaces obtained depends largely on the operator, and repeatability is limited. Furthermore, the useful stroke length is restricted by the possibilities of manual displacement of the tool post [6-8].

Another solution would also involve lowering the position of the tool tip below the rotation axis of the workpiece, but its displacement in the generatrix direction is no longer performed manually, but mechanically, by means of a guiding device or a constructive solution associated with the use of the tailstock. In this case, the movement of the tool tip is carried out more uniformly, which favors more faithful reproduction of the trajectory necessary for surface generation and the obtaining of an approximately constant roughness.

The advantages of the method consist of the better uniformity of the feed movement speed, higher productivity compared to the variant involving the use of a manual feed movement, better repeatability, and the possibility of machining longer surfaces. At the same time, the influence of the operator on the effective movement of the tool is reduced, which generally leads to approximately constant maintenance of the roughness of the machined surface. The disadvantages are related to the greater constructive complexity of the device to be used, the need for precise adjustments, the sensitivity of the system to adjustment of the tool tip position, and a lower rigidity of the technological system. For this reason, the method can be considered an intermediate solution between the simplicity of turning with manual feed and the performance offered by numerical control [6-8].

5. USING A DEVICE THAT ENSURES THE LOWERING OF THE TOOL TIP

For machining hyperboloidal surfaces on universal lathes, a device can be used that allows the surface to be generated with a lathe cutting tool by imposing an appropriate rectilinear trajectory of the tool tip. The constructive solution is based on lowering the tool tip below the rotation axis of the workpiece and displacing it in a horizontal plane parallel to this axis, along a rectilinear generatrix of the hyperboloidal surface.

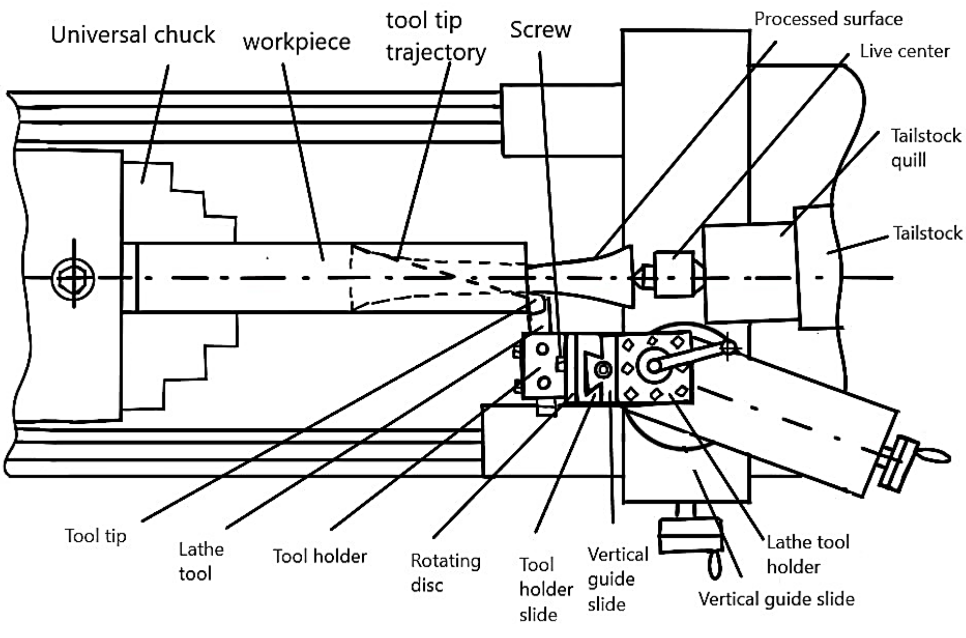

Fig. 3. The proposed device for obtaining a single-sheet hyperboloidal surface by turning, under the conditions of mounting the device on a universal lathe.

As mentioned, a lathe cutting tool is used, whose tip is positioned in a plane located below the horizontal plane of the rotation axis of the main spindle of the universal lathe, in a position parallel to the latter plane. The lathe cutting This disc can rotate around its own axis and be fixed in a tool is located and clamped by means of screws in a holder that is also attached by screws to a disc-type component. position characterized by an inclination at a certain angle with respect to a horizontal direction.

The clamping of the disc in a certain position is achieved by using two screws located in two arc-shaped channels in the disc, in diametrically opposite positions. The screws pass through threaded holes located in a slide that can be displaced along a vertical guideway attached to the tool holder of the universal lathe, a parallelepiped-shaped component attached to the vertical guideway being used for this purpose. The parallelepiped-shaped component will be located and clamped by means of the screws present in the tool post, in one of the 4 slots of the tool holder of the universal lathe.

To obtain a hyperboloidal surface, it is necessary to fix the lathe cutting tool in an inclined position in the vertical plane within its holder, by inclining and immobilizing the disc-type component at a certain angle. It is considered that this ensures a variation within acceptable limits of the values of the functional clearance and rake angles of the lathe cutting tool. This situation is due to the positioning of the tip of the lathe cutting tool in a plane located below the horizontal plane of the rotation axis of the main spindle of the universal lathe, but parallel to the latter plane, and to the displacement of the tip of the lathe cutting tool along a straight line segment belonging to a rectilinear generatrix of the future hyperboloidal surface.

The aforementioned straight line segment is also located in a plane situated below the horizontal plane of the rotation axis of the main spindle of the universal lathe, but parallel to the latter plane. For this reason, a continuous variation of the values of the functional clearance and rake angles of the lathe cutting tool will occur, as the tip of the lathe cutting tool is displaced along the straight line segment forming part of the rectilinear generator of the hyperboloidal surface to be obtained.

The tool holder of the universal lathe will be displaced with manual feed, along the direction of a generatrix of the hyperboloidal surface to be obtained on an initially cylindrical workpiece. This workpiece is located and

clamped, for example, at its left end in the universal chuck of the universal lathe, and at its right end in a rotating center placed in the quill of the tailstock of the lathe.

To ensure the manual feed displacement of the tool post along the direction of a rectilinear generator of the hyperboloidal surface to be obtained, the tool post guideway is rotated and immobilized by means of screws on the cross slide of the universal lathe.

The displacement of the tip of the lathe cutting tool along a rectilinear trajectory located in a horizontal plane situated below the horizontal plane of the rotation axis of the main spindle, and at a certain distance from this latter plane, will contribute to the materialization of a straight line segment corresponding to the generatrix of the hyperboloidal surface to be obtained. Subsequently, by rotating the generatrix around the rotation axis of the workpiece, in effect through the rotation of the workpiece, the tip of the lathe cutting tool will gradually generate a hyperboloidal surface on the workpiece.

Although the tool post guideway is rotated by an angle corresponding to the angle that the projection of the straight line segment belonging to the generator of the hyperboloidal surface makes with the rotation axis of the main spindle of the lathe, in the horizontal plane of the lathe main shaft axis, more convenient conditions for displacing the tool tip along the straight line segment acting as the generatrix of the hyperboloidal surface are obtained if the tool post can be immobilized on the tool slide such that one of its sides is parallel to the rotation axis of the main spindle of the lathe.

From a technological point of view, the analyzed

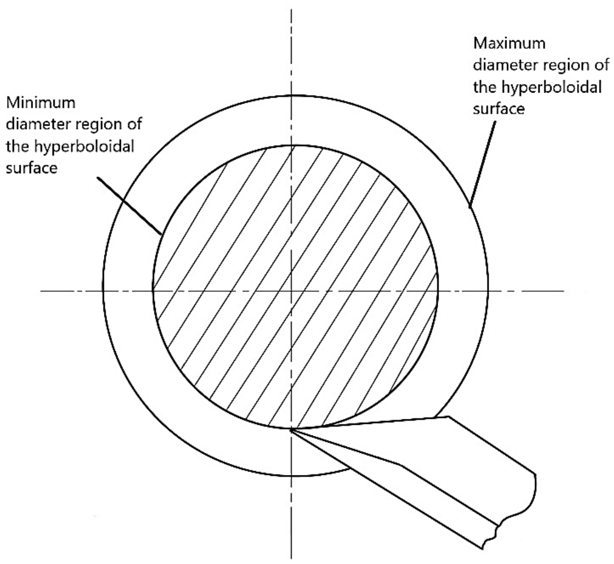

Fig. 4. Highlighting the functional values of the clearance and rake angles for two distinct positions of the lathe cutting tool tip.

solution presents the advantage of using a simple tool of the lathe cutting tool type, as well as a robust and relatively simple construction of the device. Compared to other constructive variants based on the use of milling, this solution reduces the complexity of the assembly and allows the extension of the machining possibilities of the universal lathe in the direction of obtaining hyperboloidal surfaces.

6. LIMITS OF THE PROPOSED DEVICE

Some limits of use of the proposed device result, as mentioned, from the variation of the values of the functional clearance and rake angles corresponding to the tip of the lathe cutting tool.

This variation can be followed by projecting positions of the tool tip during its displacement along the straight line segment belonging to the generatrix of the hyperboloidal surface onto a plane perpendicular to the rotation axis of the workpiece, an axis which coincides with the rotation axis of the main spindle of the lathe (Fig. 4).

It can be observed that if, upon entry of the tool tip into the workpiece material, the functional clearance angle has a value α_1, and for the rake angle γ_1 a null value has been conventionally adopted, then when the tip of the lathe cutting tool reaches a position situated below the rotation axis (a position to which the smallest value of the diameter of the hyperboloidal surface of revolution will correspond), the value of the functional clearance angle α_2 decreases, while the value γ_2 of the functional rake angle increases.

Subsequently, as the tool tip moves further from the vertical plane containing the rotation axis of the workpiece, the value of the functional rake angle γ will continue to increase, while the value of the functional clearance angle α will continue to decrease, which could lead to entirely unfavorable aspects from the point of view of the durability of the lathe cutting tool. For this reason, the zone corresponding to the decrease in diameter of the hyperboloidal surface of revolution may be obtained under relatively favorable cutting conditions, while the continuation of the turning process in the zone where the diameter begins to increase may no longer ensure acceptable cutting conditions.

This problem could lead to the solution of turning the hyperboloidal surface such that obtaining the minimum diameter requires two locations and fixings of the workpiece. In the first clamping, the zone in which the diameter decreases could be obtained, after which the workpiece would be repositioned in the reverse position so that the other branch of the hyperboloidal surface could be obtained under similar cutting conditions. However, this solution presents a high risk of failing to ensure the coaxiality of the two zones of the desired hyperboloidal surface, it being well known that the universal chuck does not ensure high positional accuracy when two distinct fixings are used. If the machining carried out in a single phase, namely the phase in which the diameter of the surface decreases continuously, may be considered acceptable in the case of bevel-hyperboloidal gear wheels, the same approach is no longer appropriate when the objective is to produce cylindrical-hyperboloidal gear wheels or rollers intended for the straightening of cylindrical bars.

7. CONCLUSIONS

In the paper, the problem of obtaining single-sheet hyperboloidal surfaces by turning was addressed. Such surfaces are encountered in industrial practice, for example, in the case of hyperboloidal rollers used for straightening rectilinear bars, or in the case of workpieces from which the gear wheels of a bevel-hyperboloidal gear are to be obtained. The study of the specialized literature highlighted the possibilities of obtaining single-sheet hyperboloidal surfaces on numerically controlled machine tools, but also by using specialized milling devices.

The analysis of these devices led to the idea of seeking a solution that would allow single-sheet hyperboloidal surfaces to be obtained on universal lathes.

A device was gradually outlined that would allow the use of lathe cutting tools on a universal lathe. However, the analysis of the cutting conditions corresponding to the lathe cutting tool also highlighted certain limitations on the possibilities of using the proposed device.

It is anticipated that research will continue through the materialization of such a device and the clarification of the conditions of its use.

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