Proceedings of the 16th International Conference on Manufacturing Systems – ICMaS
Published by Editura Academiei Române, ISSN 1842-3183

"Politehnica" University of Bucharest, Machine and Manufacturing Systems Department
Bucharest, Romania, 22 November, 2007

CONSTRUCTION AND DIRECT KINEMATICS MODEL OF PARALLEL ROBOTS

Laurentiu NAE, Cristina IBRAIM, Adrian OLARU, Nicolae PREDINCEA

Abstract: *One aspect of this article is to improve the absolute accuracy of these systems by means of calibration techniques. This is to develop algorithms which adapt the initially perfectly regarded geometric parameters of the transformation equations relating joints coordinates to world coordinate to real robot's structure. The parallel robots are used for fast and accurate positioning for fulfilling the increasing requirement in handling and assembly.*

Key words: *parallel robot, joint, coordinate system, degree-of-freedom.*

1. INTRODUCTION IN PARALLEL ROBOTS

1.1. Parallel robots versus serial robots

For industrial robots, there are generally two main types of the manipulators: serial manipulators and parallel manipulators, a parallel robot is a closed-loop mechanism in which the mobile platform is connected to the base by at least two serial kinematical chains (legs). Applications of this type of robots can be found in the motion platform for the pilot training simulators and the positioning device for high precision surgical tools because of the high force loading and very fine motion characteristics of the closed-loop mechanism. Recently, researchers are trying to utilized these advantages to develop parallel-type robot based multi-axis machining tools and precision assembly tools.

Conversely, they suffer from smaller work volume, singular configurations and a more complicated direct kinematical solution (which is usually not required for control purposes) [1].

Unlike parallel robots (Fig. 1), a serial robot (Fig. 2) is an open-ended structure consisting of several links connected in series. The human arm is a good example of a serial manipulator. Presently, all the developed manipulators have more or less the same shapes. As they

Fig. 1. Delta parallel robot.

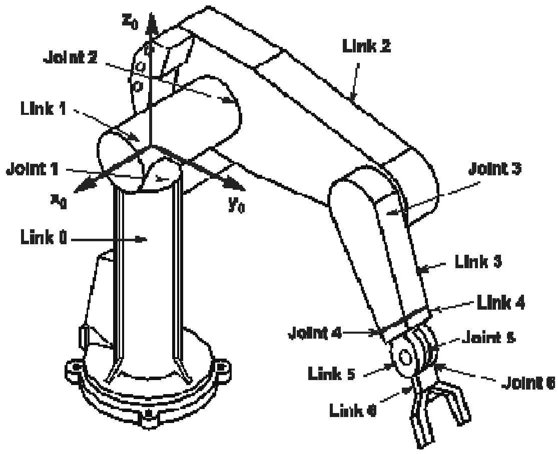


Fig. 2. Vertical knick arm robot.

are well-constructed machines, hence are often used in the industrial applications. However, as the actuator in the base has to carry and move the whole manipulator, with its links and actuators, hence it is a well-known fact that it is very difficult to realize very fast and highly accurate motions with such manipulators. As a consequence, there arise the problems of bad stiffness and reduced accuracy.

Based on the fact that the end-effector's position can be defined by a point in space and that its orientation with two degrees of freedom can be described by a line from a first point to a second point in space, thus forming a joining element. It is clear that an end-effector with 5 degrees of freedom can be described by means of two points in space. Should six degrees of freedom be desired, then three points in space are necessary.

However, the increasing interest in parallel robots points to the potential embedded in this structure, which has not been yet fully exploited. The advantages of parallel robots as compared to serial ones are:

- higher pay-load-to-weight ratio since the payload is carried by several links in parallel;
- higher accuracy due to non-cumulative joint error;
- location of motors at or close to the base;
- simpler solution of the inverse kinematics equations.

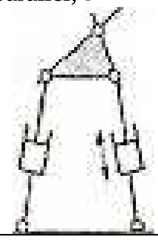
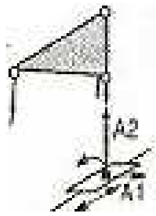
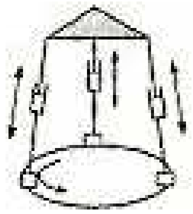
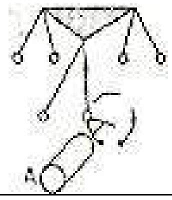
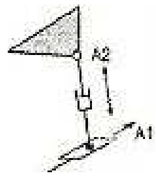
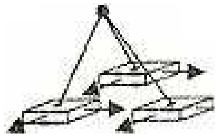
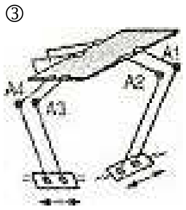
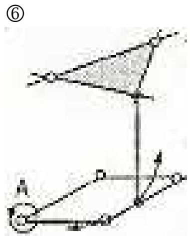
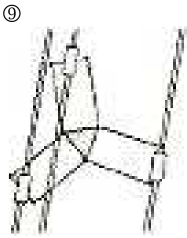
• higher structural rigidity, since the load is usually carried by several links in parallel;①  Hexapod Stewart Platform (1965)	④  Combining translational and rotary motion with fixed-length strut from Kholi (1988)	⑦  Circular movement of base point from Alizade (1992)
②  from Hunt (1983)	⑤  Combination of linear driven base point and variable strut length motion from Behi (1988)	⑧  Technion Haifa (1994)
③  Parallelogram and revolute joints, deltapatent from Clavel (1985)	⑥  Translation from the base point with parallelogram from Han (1989)	⑨  Linear delta robot from Clavel (1994)

Fig. 3. Known hexapod systems.

All three joints of an arm element can be motor driven but it is also possible to motorise only two or even just a single joint. The number of arm elements required for defined motion of an end-effector (or platform) for a given number of degrees of freedom, is dependent on the number of motorised (active) joints that each arm element possesses.

Fig. 3 shows some of the many possible configurations of parallel link machines. The development of parallel link kinematics is not new but has been intensively researched during the last two decades.[3]

All of the existing machines share one common characteristic – motion is generated by either arm length modification, positioning of the base points or a combination of both. If the position of a point in space is to be described by the end position of an arm element, then this arm element must have three degrees of freedom [5]. In order to realise this three degrees of freedom, the following possibilities present themselves:

– change in arm length;
– movement in arm base point;
– rotation around the arm base point.

These three types of motion result in six possibilities for arm kinematics as shown in Fig. 4.

1. Two kinematics with a single translational and two rotary joints.
2. Two kinematics with a rotary and two translational joints.
3. One kinematics with three translational joints.
4. One kinematics with three rotary joints.

Kinematics for freely definable points in space

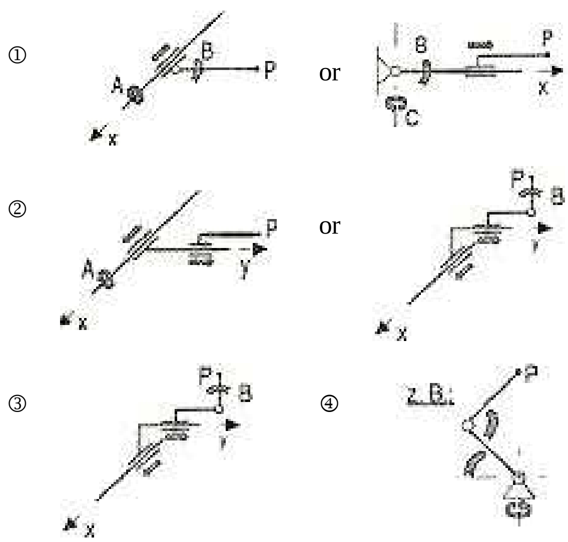
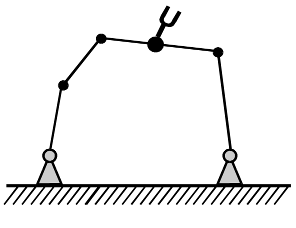
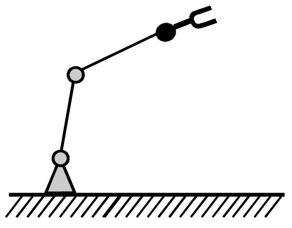
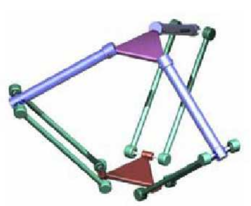
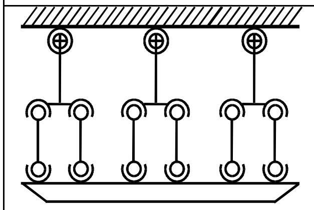
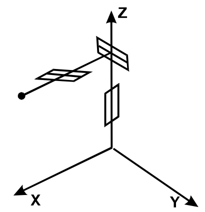


Fig. 4. All possible configurations of arms with 3 DOF.

Parallel robots belong to the closed change mechanism because they start and end at the base. In other words a design model for a parallel robot would only be the effort of designing one chain which is usually repeated symmetrically for the whole robot. On the other hand the design model of a serial mechanism is usually more complicated since its links are subjected to bending forces

Table 1

Comparison between parallel and serial robots

Parallel robots	Serial robots
	
	
	
• High payload-to-weight ratio • High mechanical rigidity • Low moving mass • Higher accuracy, no cumulative error • Simple design of Kinematical chains • Limited workspace • Adapted to specific applications	• Low payload-to-weight ratio • Low mechanical rigidity • High moving mass • Joint cumulative error (lesser accuracy) • Complex design of components • Large workspace • General purpose robot

as well, which makes the design more complicated to insure the stiffness of the mechanism.

Table 1 shows a clear comparison between parallel and serial structure robots. Note in the first 2D structure the difference between the closed chain mechanism and the open chain mechanism.

A kinematical representation of both mechanisms can also be shown with respect to the type of joints connecting each link to the other [4]. However, the complicate structure of the parallel mechanism not only limits the motion of the platform but also creates complex kinematical singularity in the workspace of the mobile platform, and therefore, makes the design, trajectory planning and application development of the parallel robot difficult and tedious.

1.2. Kinematical representation of Parallel robots

Kinematical representation of parallel robots shows in a simple way the kinematical structure of a parallel robot. Not all types of robots can be easily represented cinematically since there are other asymmetrical parallel robots with special applications which can not be represented cinematically like other symmetrical structures. Asymmetrical robots usually have their joints and mechanical chains distributed in a non-homogeneous manner for example in different plans and at different orientation angles, which makes it hard to define the structure of the robot.

Table 2

Types of joints

	Passive Joints
D_3	Spherical joint
D	Revolute joint
D_2	Universal joint
SS	Prismatic joint
DS	Rotation/Translation joint
	Active Joints
$D3_a$	Spherical joint
D_a	Revolute joint
$D2_a$	Universal joint
S_a	Prismatic joint
DS_a	Rotation/Translation joint

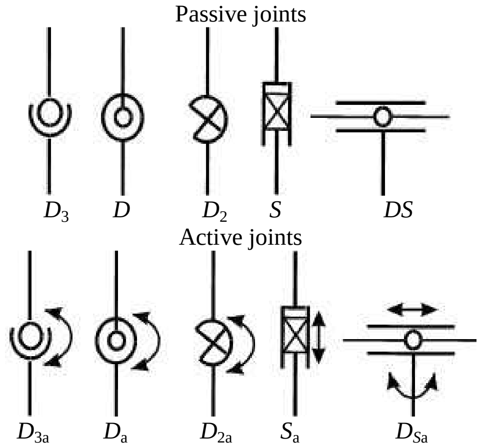


Fig. 5. Main different types of joints used in parallel robots.

Compared to the clasical machine tool, the kinematics of the parallel manipulator is much more complex.

In general, the kinematics includes two aspects:

– forward kinematics;

– inverse kinematics.

Of particular interest here is that, whereas in serial mechanism, the forward kinematics problem is easy and the ineverse kinematics problem is challenging, the converse is true of parallel mechanism.

Mainly parallel robots can be represented with respect to the structure of their parallel mechanism. A parallel mechanics is a repetition of number of mechanical chains which are connecting the platform to the base.

There are two types of joints: passive joints and active joints as they are presented in Table 2.

Therefore, it is important to understand the notation of the different joints used which connect the links together to form the mechanical chain. (Table 3)

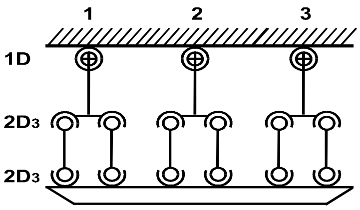
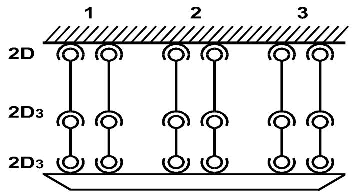
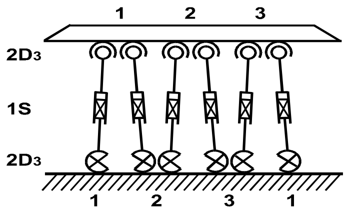
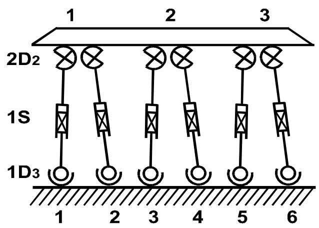
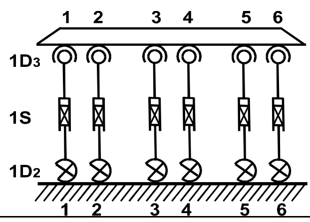
Fig. 5 shows the main different types of joints used in parallel robots to connect links together and form a mechanical chain.

2. DIRECT KINEMATICAL PROBLEM (DKP)

In recent years, a number of methods have been developed for the kinematical analysis of robot arms and mechanical manipulators. Nevertheless, the applications

Table 3

Notation of different types of joints

Name/Formula/DOF	Kinematical structure diagram	Description / each stage
Delta robots **3 (1, 2, 2) D D3 D3** **3 DOF**		**Fixed base** **3(1)** active revolute joints **3(2)** passive spherical joints **3(2)** passive spherical joints **Platform**
Delta robot **3 (2, 2, 2) D D3 D3** **6 DOF**		**Fixed base** **3(2)** active revolute joints **3(2)** passive spherical joints **3(2)** passive spherical joints **Platform**
Hexapod type 3-3 **3 (2, 1, 2) D3 2S D2** **6 DOF**		**Platform** **3(2)** passive spherical joints **6(1)** active prismatic joints **3(2)** passive universal joints **Fixed base**
Hexapod type 3-6 **3 (1, 1, 2) 2D3 2S D2** **6 DOF**		**Platform** **3(2)** passive spherical joints **6(1)** active prismatic joints **3(2)** passive universal joints **Fixed base**
Hexapod type 6-6 **6 (1, 1, 1) D3 S D2** **6 DOF**		**Platform** **3(2)** passive spherical joints **6(1)** active prismatic joints **3(2)** passive universal joints **Fixed base**

of most of these methods are restricted to only serial robots. The few methods which deal with parallel robots (i.e. the robots with a combined closed loop and open chain structure) are also limited to specific robots with simplified structures. However, as the applications of parallel robots become more popular, and their structures become more complex, it is essential to have a systematic and efficient numerical method for analyzing the kinematical characteristics of general parallel robots.

One difficulty in analyzing parallel robots is that the driving mechanisms of the robot may contain many multi-degree-of-freedom (DOF) joints and several coupled, closed kinematical loops. Thus the local coordinate systems can not be assigned sequentially as with conventional serial robots. In addition, the displacement (or rotation) of the joint variables are constrained by the loop closure conditions.

The first sections of this paper deal with recursive coordinate transformation.

The other sections of this paper deal with the displacement analysis. A set of recursion formulae is used for efficient forward coordinate transformations. These formulae are derived based on the Rodrigues' formula for spatial rotation, and can be extended to handle various types of multi-DOF joints. A two-phase numerical algorithm for displacement analysis of general parallel robots is presented here. In the first phase of the algorithm, the displacement analysis problem is formulated as an optimization problem. A generalized cyclic coordinate descent (CCD) method is used for finding a good approximation of the solution vector. The second phase of the algorithm is based on the iterative method for displacement analysis of linkages.

It should be mentioned that there are certain available commercial software packages, such as ADAMS, DADS and SIMPACK, which may also be used for the numerical kinematical analysis of general parallel robots [2].

The algorithm used in this work is based on the relative coordinate formulation and the direct application of the loop closure conditions. Hence, the size of the data structure and the required computations are significantly reduced. Consequently, it can be executed efficiently on small computers, such as a personal computer.

3. FORWARD COORDINATE TRANSFORMATION

Once the structure has been defined, the coordinate systems attached to the robot can be defined as follows. Each link is attached with l_d local coordinate systems, where is the total number of joints incident to the link.

The origins of the coordinate systems are located at the centres' joints with the unit vectors along the coordinate axes denoted as X_j, Y_j, Z_j if joint j is an outlet joint of the link, and as U_j, V_j and W_j if it is an inlet joint.

Therefore, each joint of the robot is associated with two coordinated systems, since if it is an outlet joint of one link then it must also be an inlet joint of another link.

4. LINK TRANSFORMATION METHOD

The relationship between coordinate systems (U_i V_i W_i) and (X_j Y_j Z_j) attached to link l is shown in Fig. 6 Here, T_{ij} is a unit vector along the common normal line between axis W_i and Z_j and is directed from W_j to Z_j ; a_{ij} is the signed distance from W_i to Z_i and α_{ij} is the angle between W_i and Z_i measured counter clockwise (ccw) about T_{ij}.

Similarly, b_{ij} is the signed distance from T_{ij} to X_j, and β_{ij} is the angle between T_{ij} and X_j measured ccw about Z_j.

Finally, c_{ij} is the signed distance from U_i to T_{ij} and γ_{ij} is the angle between U_i and T_{ij} ccw about W_i. The six constant parameters a_{ij}, b_{ij}, c_{ij}, α_{ij}, β_{ij}, γ_{ij} are referred to the *shape parameters* of the two coordinate systems.

Based on the Rodrigues' formula for spatial rotation and the shape parameters defined previously,
the orientation and position of coordinate system (X_i Y_i Z_i) with respect (U_i V_i W_i) can be computed by using the following steps:

Step 1: Obtain T_{ij} by rotating U_{ij} about W_i with angle γ_{ij}. Noting that $V_i = W_i \times U_i$, and $U_i \cdot W_i = 0$, we have

$$T_{ij} = U_i \cos \gamma_{ij} + V_j \sin \gamma_{ij}. \qquad (5.1)$$

Step 2: Obtain Z_j by rotating W_i about T_{ij} with angle α_{ij}. Since $T_{ij} \cdot W_j = 0$, thus

$$Z_j = W_i \cos \alpha_{ij} + (T_{ij} \times W_i) \sin \alpha_{ij}. \qquad (5.2)$$

Step 3: Obtain X_j by rotating T_{ij} about Z_j with angle β_{ij}. Since $T_{ij} \cdot Z_j = 0$, thus

$$X_j = T_{ij} \cos \beta_{ij} + (Z_j \times T_{ij}) \sin \beta_{ij}. \qquad (5.3)$$

Step 4: Y_i is simply the vector cross product of Z_i and X_i thus

$$Y_j = Z_j \times X_j. \qquad (5.4)$$

Step 5: Compute the position vector P_{ij} from

$$P_{ij} = c_{ij} W_{ij} + a_{ij} T_{ij} + b_{ij} Z_j . \qquad (5.5)$$

These expressions have a recursive character and they represent the base for the transformation of the coordinate.

5. JOINT TRANSFORMATION METHODS

The recursive relation of the coordinate systems between two neighbouring links can also be easily derived by using the Rodrigues' formula. For instance, the common characteristic revolute, prismatic and cylindrical joint is that they only have one joint axis to allow relative motions between the jointed links. The joint axis is thus conveniently aligned with the Z_i and the W_j axes, as shown in Fig. 6.

The recursive relation between the two coordinate systems (U_j V_j W_j) and (X_j Y_j Z_j) can be obtained as:

$$W_j = Z_j,$$
$$U_j = X_j \cos \theta_j + Y_j \sin \theta_j,$$
$$V_j = W_j \times U_j,$$
$$P_j = S_j \cdot Z_j.$$

Where θ_j is the angle between axes X_j and U_j, measured ccw about Z_j, and S_j is the signed distance from X_j to U_j measured along Z_j. Noting that if joint j is a cylindrical joint, then both θ_j and S_j are the joint variables. If joint j is a revolute joint, then θ_j is the joint variable and S_j is equal to zero. The reverse is true if it is a prismatic joint.

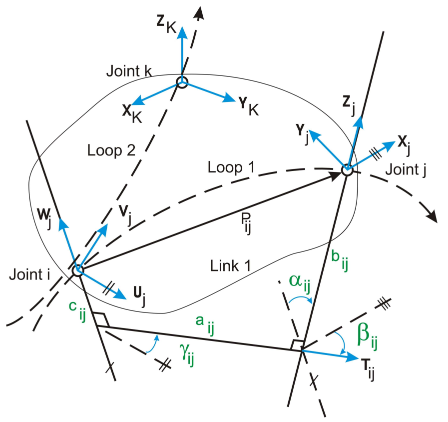


Fig. 6. Definition of local coordinate systems.

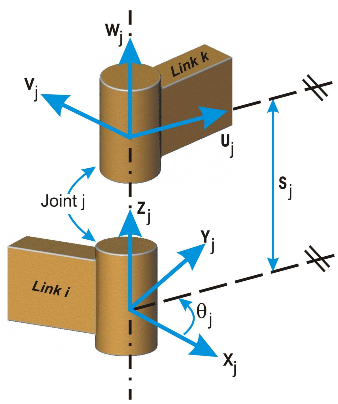


Fig. 7. Revolute, prismatic and cylindrical joints.

Table 4

Joint variables of multi-DOF joints

Recursion Formulae and Variables of Multi – DOF Joints	
Universal Joint	**Spherical Joint**
$W_j = X_j \cos\theta_{j1} + Y_j \sin\theta_{j1}$ $U_j = X_j \cos\theta_{j2} + (W_j \times Z_j)$ $\sin\theta_{j2}$ $V_j = W_j \times U_j$	$R_j = Z_j \cos\theta_{j2} + Y_j \sin\theta_{j1}$ $W_j = Z_j \cos\theta_{j2} + (R_j \times Z_j) \sin\theta_{j2}$ $U_j = R_j \cos\theta_{j3} + (W_j \times Z_j) \sin\theta_{j2}$ $V_j = W_j \times U_j$
Joint Variable	
θ_{j1}, θ_{j2}	$\theta_{j1}, \theta_{j2}, \theta_{j3}$

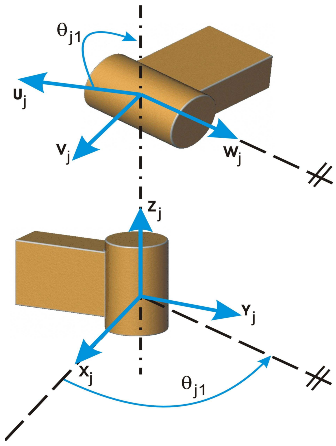


Fig. 8. Universal joint.

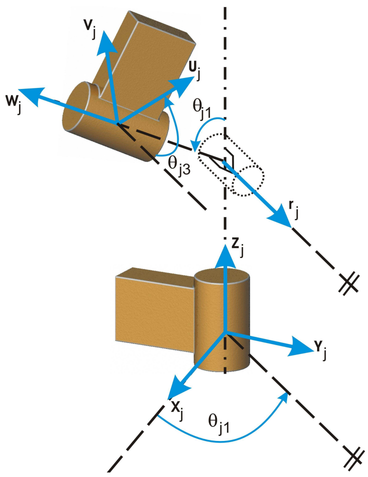


Fig. 9. Spherical joint.

By using the same methodology, the recursion formulae for other types of joints can also be derived, since most of them can be considered as combinations of the revolute and prismatic joints. For example, a Hooke-type universal joint can be considered as two perpendicularly intersecting revolute joints and a spherical joint can be modelled as three mutually orthogonal intersecting revolute joints, as shown in Figs. 8 and 9 respectively. The recursion formulae and the joint variables for these joints are given in Table 4.

6. CONCLUSIONS

Usually the parameter identification process leads to systems of nonlinear equation which need to be solved. By reducing the number of parameters, convergence rate of this process may be improved. However, due to highly nonlinear relationships it is not possible for parallel robots to determine if there are parameter variations, which may be neglected because of minor influence on the absolute pose accuracy. A simulation system shall be developed, which is intended to determine the influence of geometric parameter variation on the absolute accuracy of parallel robots.

7. REFERENCES

[1] Kevin, L., Conrad, P., Shiakolas, S. (2000). *Robotic calibration issues: Accuracy, Repeatability and Calibration*, Mediterranean Conference on Control & Automation, Greece.

[2] Li-Chun, W., Cheng Chen, C. (1993). *On the Numerical Kinematic Analysis of general Parallel Robotic Manipulators*, IEEE Trans. Robotics Automat.

[3] Nanua, P., Waldron, K.J., Murthy, V. (1990). *Direct kinematics solution of a Steward platform*, IEEE Trans. Robotics Automat. Vol. 6, pp. 438-444.

[4] Szatmari, S. (1999). *Geometrical errors of parallel robots*, Oeriodica Polytehnica Ser. Mech. Eng., Vol. 43, pp. 155-162.

[5] Zhuang, H., Liu, L. (1996). *Self Calibration of a Calss of Parallel Manipulator*, International Conference on robotics and automation Minneapolis, Minesota.

Authors:

Eng, Laurenţiu NAE, ADACOMPUTERS S.R.L.,
E-mail: laurentiu.nae@adacomputers.ro
Eng, Cristina IBRAIM, Renault Technologie Roumanie S.R.L., Concepteur CAO,
E-mail: cristina.ibraim@renault.com
Ph.D, Adrian OLARU, Professor, University "Politehnica" of Bucharest, Machine and Manufacturing Systems Department,
E-mail: aolaru_51@yahoo.com
Ph.D, Nicolae PREDINCEA, Professor, University "Politehnica" of Bucharest, Machines and Production Systems Department,
E-mail: predi@imst.msp.pub.ro